# Finite Sample Evidence on the Performance of Stochastic Frontier Models Using Panel Data* 

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#### Abstract

Most stochastic frontier models have focused on estimating average productive efficiency across all firms. The failure to estimate firm-specific effiicency has been regarded as a major limitation of previous stochastic frontier models. In this paper, we measure firm-level efficiency using panel data, and examine its finite sample distribution over a wide range of the parameter and model space. We also investigate the performance of the stochastic frontier approach using three estimators: maximum likelihood, generalized least squares and dummy variables (or the within estimator). Our results indicate that the performance of the stochastic frontier approach is sensitive to the form of the underlying technology and its complexity. The results appear to be quite stable across estimators. The within estimator is preferred, however, because of weak assumptions and relative computational ease.


## 1. Introduction

Since the pioneering work of Farrell [1957], ${ }^{1}$ a large literature has developed on measuring productive efficiency. The majority of studies have dealt with two basic issues. The first is how to define the production function of a firm or an industry, and the second is how to measure efficiency. The two questions are closely related since the production function is used as a yardstick for efficiency measurement.

We have some theoretical guidance for the first issue. A production function is defined to be the maximum possible output quantity, given inputs and the state of technology. Such a function is called a frontier production function or a best practice technology, as opposed to the traditional estimated average production function or average practice technology. Accordingly, we choose a frontier instead of an average production function as a reference point to measure efficiency. ${ }^{2}$

Even though there exists some theoretical guidance to measure productive efficiency [Debreu 1951; Farrell 1957; Färe, Grosskopf and Lovell 1985], ${ }^{3}$ efficiency

[^0]measurement mostly depends on empirical or experimental studies. Efforts to measure efficiency can be divided into two approaches-statistical and nonstatistical [Schmidt 1985; Charnes and Cooper 1985]. Although neither approach can give a definitive answer to the way in which efficiency is measured, we use the stochastic frontier approach as representative of the statistical approaches in this paper. The justification is that stochastic frontier models are based on more reliable assumptions and empirical results than a full frontier [Greene 1980a, b] model. We will leave nonstatistical approaches to a future study.

Stochastic frontier models rely on two important assumptions: The specific functional form to characterize the underlying technology, and the form of the stochastic components. The error terms in a stochastic frontier model usually contain a symmetric random error to capture noise and exogeneous shocks, and a one-sided error term to allow for inefficiency such as managerial related mistakes [Aigner, Lovell and Schmidt 1977; Meeusen and van den Broeck 1977].

Previous stochastic frontier approaches largely have focused on estimating the average efficiency of all firms in an industry. Exceptions include studies by Jon drow et al. [1982], Huang and Bagi [1984], Schmidt and Sickles [1984], Waldman [1984], Cornwell, Schmidt and Sickles [1989], and Battese and Coelli [1988]. Efficiency measurement, which is based on cross sectional data, is of an average nature across all firms and fails to identify firm-specific inefficiency. This has been regarded as a major limitation of previous stochastic frontier models. ${ }^{4}$ However, there are two other serious drawbacks. The first is that a specific form for the distribution of productive inefficiency is usually assumed in order to identify average inefficiency. This means that estimation of productive inefficiency can be sensitive to a priori assumptions on the distribution of productive inefficiency. The second is that inefficiency is often assumed to be independent of the inputs. Intuitively, this does not correspond to the behavioral assumptions of firms. A firm, deciding on its choice of inputs, conditions its decision on the information set, which may include the perceived distribution of efficiency within the industry. We hope to show that these common problems can be ameliorated in part by a stochastic frontier approach using panel data.

There are a few papers which compare results from the application of different frontier models to the same data [Kopp and Smith 1980]. Different measurement methods have also been compared for the same data, Banker, et al. [1986], Banker, Conrad and Strass [1986], Nelson and Waldman [1986], Bauer et al. [1986]. The common findings of these studies show that efficiency measurement depends on the choice of functional forms to approximate the underlying technology, and on the estimation methodologies employed. Unfortunately, further analysis of these findings are hindered by the lack of knowledge of the true production and inefficiency structures. To address this limitation we utilize Monte Carlo techniques which allow us to control the structure of the underlying technology and firmlevel inefficiency. In our experiments we use approximating functional formsthe CES-translog (CES-TL), the translog (TL) and the generalized Leontief (GL)-to estimate firm-specific inefficiency. We examine the robustness of estimated firm-specific technical inefficiencies using three estimation techniques:
(1) maximum likelihood (mle), (2) generalized least squares (gls) and (3) the within estimator.

The organization of this paper is as follows. Section 2 explains how the stochastic frontier model uses panel data for inefficiency measurement. Section 3 outlines three estimators-mle, gls and the within estimator. Section 4 contains the general design of the Monte Carlo study. In Section 5, we estimate with a nonflexible function (CES) firm-specific technical inefficiency using information about known arbitrary production technologies. We also investigate, in the same section, firmlevel inefficiency using three flexible functional forms. Finally, we discuss conclusions and a direction for future research.

## 2. The model

To fix ideas about productive efficiency, we regard a firm to be a cost-minimizer. If a firm achieves its goal in a production activity, it is called an economically efficient firm; if a firm cannot attain its objective, it is called an economically inefficient firm. Following Farrell, a firm may fail to minimize the cost of producing its output in two ways: (1) it may be technically inefficient, failing to operate on the production frontier or cost frontier or (2) it may be allocatively inefficient, failing to employ the least cost mix of inputs given the fixed relative input prices. This study focuses only on the former source of inefficiency. ${ }^{5}$

Under the assumption that all firms in an industry have the same deterministic production process, ${ }^{6}$ the representative firm's stochastic production function can be written as:

$$
\begin{align*}
y_{i t} & =f\left(x_{i t}, \beta\right)+\varepsilon_{i t} \\
\varepsilon_{i t} & =v_{i t}-u_{i} \\
u_{i} & \geqslant 0 \\
i & =1, \ldots, N \\
t & =1, \ldots, T \tag{1}
\end{align*}
$$

where $y_{i t}$ is an observation on output, $x_{i t}$ is a vector if obsrvations on inputs, $\beta$ is a vector of unknown production paramters, and $\varepsilon_{i t}$ is a random disturbance composed of a symmetric white noise component $v_{i t}$ and a nonnegative component $u_{i}$ which represents firm specific technical inefficiency. Let $\mathbf{N}$ represent the number of firms, and $T$ the number of time periods. This is a panel data model which is a cross section of firms (or plants), each observed for a number of time periods. ${ }^{7}$

In order to estimate (1), we need to first specify $f\left(x_{i t}, \beta\right)$. The chosen functional form depends largely on a priori information about the underlying technology. Without specific engineering blue prints, etc., the choice of functional form is
usually based on its flexibility [Diewert 1971; Gallant 1981]. ${ }^{8}$ We consider the CRESH (Constant Ratio of Elasticity of Substitution, Homothetic) production function, which was introduced by Hanoch [1971] and used by Guilkey and Lovell [1980] and Guilkey, Lovell and Sickles [1983] as the underlying flexible technology.
Let us suppose a firm utilizes $m$ inputs $X_{i t}=\left(x(1)_{i t}, \ldots, x(m)_{i t} \geqslant 0\right.$ to produce a single output $\mathrm{y}_{\mathrm{it}} \geqslant 0$ for all $\mathrm{i}, \mathrm{t}$ with the production technology represented by

$$
\begin{equation*}
\mathrm{y}_{\mathrm{it}} \mathrm{e}^{\mathrm{yy}_{\mathrm{it}}}=\left(\sum_{\mathrm{k}=1}^{\mathrm{m}} \delta_{\mathrm{k}} \mathrm{x}(\mathrm{k})_{\mathrm{it}^{-p_{k}}}\right)^{-\gamma / \mathrm{p}} \tag{2}
\end{equation*}
$$

where $\theta \geqslant, \gamma>0, \delta_{k}>0$, for all $k, \sum_{k=1}^{m} \delta_{k}=1$.
The advantage of (2) is that it nests many well known deterministic production functions under certain parametric restrictions. For example, if $\theta=0$, (2) is almost homogeneous CRES, if $\rho_{1}=\ldots=\rho_{m}$, (2) is homothetic CES (Constant Elasticity of Substitution), if $\theta=0$ and $\rho_{1}=\ldots=\rho_{m}$, (2) is homogeneous CES, and it has homothetic Cobb-Douglas as a limiting form as $\left(\rho_{1}=\ldots=\rho_{m}\right) \rightarrow 0$. Under further restrictions, (2) can represent the Cobb-Douglas production function and the CES production function. In other words, under $\theta=0, \rho_{1}=\ldots=\rho_{m}=\rho$ and $\gamma$ $=1$, (2) becomes the CES production function. If $\theta=0,\left(\rho_{1}=\ldots=\rho_{m}=\rho\right) \rightarrow 0$ and $\gamma=1$, then the limiting form of (2) is the Cobb-Douglas production function.
To represent the complexity and structure of the underlying technology, we use returns to scale, $\gamma(\mathbf{x})$ and Allen-Uzawa partial elasticities of substitution (AES) between inputs, $\sigma_{i j}(x),{ }^{,}$

$$
\begin{align*}
& \gamma(x)=\frac{1}{1+\theta y} \cdot \frac{\Sigma \rho_{k} \delta_{k} x_{(k)}}{\Sigma \rho \delta_{k} x(k)^{-\rho_{k}}},  \tag{3}\\
& \sigma_{i j}(x)=\frac{1}{\left(1+\rho_{i}\right)\left(1+\rho_{j}\right)} \cdot \frac{\Sigma \rho_{k} \delta_{k} x(k)^{-\rho_{k}}}{\Sigma\left\{\left(\rho_{k} \delta_{k}\right) /\left(1+\rho_{k}\right)\right\} \cdot x(k)^{-\rho_{k}}}, \text { for } i \neq j  \tag{4}\\
& \sigma_{i i}(x)=\frac{1}{\left(1+\rho_{i}\right)} \cdot \frac{\Sigma \rho_{k} \delta_{k} x(k)^{-\rho_{k}}}{\rho_{i} \delta_{i} x(i)^{-\rho_{i}}\left(1+\rho_{i}\right)} .  \tag{5}\\
& -\left[\Sigma\left\{\left(\rho_{k} \delta_{k}\right) /\left(1+\rho_{k}\right)\right\} x_{k}^{-\rho_{k}}-\left\{\left(\rho_{i} \delta_{i}\right) /\left(1+\rho_{i}\right)\right\} x(i)^{-\rho_{i}}\right] \\
& \Sigma\left\{\left(\rho_{k} \delta_{k}\right) /\left(1+\rho_{k}\right)\right\} x(k)^{-\rho_{k}}
\end{align*},
$$

where we have omitted the observation subscripts for notational clarity.
We next turn to the stochastic portion of (1). As has been noted by Aigner et al. [1977], the first one of two disturbances, $\mathrm{v}_{\mathrm{it}}$ is typically assumed to have a symmetric distribution about zero and represents the usual statistical noise due to such
factors as luck, climate, topography, and machine performance. The second disturbance is assumed to have a non-negative distribution which reflects technical inefficiency. Even though we have no a priori knowledge that the distribution of technical inefficiency has a specific non-negative form, we can examine different non-negative distributions to see how robust our estimates of productive efficiency will be. We will assume in this study that technical inefficiency is time invariant. ${ }^{10} \mathrm{~A}$ justification is that firm-specific inefficiency can be regarded as an inherent or structural residual between observed data and the corresponding production (or cost) frontier. Without violent changes in economic environments (that is, deregulation), firm-specific efficiency and its relative ranking are not likely to change drastically over finite time periods.

The advantage of a stochastic frontier model using panel data is placed on the estimation of the mean value of $u_{i}, E\left(u_{i}\right)$. The residual, $v_{i}-u_{i}$, of the usual stochastic frontier models using cross sectional data contains noise ( $v_{i}$ ) and cannot be used as a measure of $u_{i}$. Recently, a better measure, $E\left(u_{i} \mid v_{i}-u_{i}\right)$, was suggested by Jondrow et al. and Waldman. This measure is dependent on distributional assumptions, and still is contaminated by the presence of $\mathrm{v}_{\mathrm{i}}$. With panel data we can estimate $u_{i}$ unconditionally, because we get to observe it T times instead of once. Intuitively, we are just averaging away the noise $\mathrm{v}_{\mathrm{it}}$ over a large number of time periods.

Since the expected value of the one-sided distribution is nonzero, ${ }^{11}$ it is necessary for us to correct skewedness in order to have unbiased estimates of (1). The corrected model is written as

$$
\begin{equation*}
y_{i t}=(-\mu)+f\left(x_{i t}, \beta\right)+\left(\mu-u_{i}\right)+v_{i t} \tag{6}
\end{equation*}
$$

where $\mu$ is equal to $E\left(u_{i}\right)=1 / T \sum_{i=1}^{T} u_{i}$; and

$$
\begin{equation*}
y_{i t}=(-\mu)+f\left(x_{i t}, \beta\right)-u_{i}^{*}+v_{i t}, \tag{7}
\end{equation*}
$$

where $u_{i}^{*}$ is equal to ( $u_{i}-\mu$ ), so that the error terms $v_{i t}$ and $u_{i}^{*}$ have zero mean.

It is well known that either the cost function or the production function uniquely define the true production technology. ${ }^{12}$ Thus we also may estimate firmspecific inefficiency using a stochastic frontier cost function. The choice between production versus cost function analysis should be based on exogeneity assumptions concerning the input levels, on the one hand, and the level of output, on the other. In practice the choice is usually made on the basis of available data and computational convenience. When using cost frontiers insteads of production frontiers, the same arguments can be employed as above, except that the two error-components change sign.

## 3. The estimators

We introduce three estimators for the stochastic frontier model using panel data. For each estimator, we clarify the basic assumptions and then explain how to derive the estimators. Finally, we form an index of firm-specific technical inefficiency, which is based on the difference between the estimated production or cost frontiers, and the observed data.

As an estimator in previous stochastic frontier approaches, ${ }^{13}$ the use of maximum likelihood requires several strong assumptions. First, we assume that the regularity conditions for the density functions hold [Norden 1972; 1973]. Second, we assume that $u_{i}$ and $v_{i t}$ are half-normal and normal random variables. Third, technical inefficiencies are assumed to be independent of inputs. The likelihood function based on these assumptions was derived by Pitt and Lee [1981]. We briefly mention the basic procedure of derivation [Schmidt and Sickles 1984].

From (7), $\mathbf{u}_{\mathbf{i}}$ is identical independently distributed (i.i.d.) with the half-normal density function,

$$
\begin{equation*}
\mathbf{h}(u)=2 / \sqrt{2 \pi \sigma_{u}^{2}} \cdot \exp \left\{-\mathbf{u}_{\mathbf{i}} / 2 \sigma_{u}^{2}\right\}, \mathbf{u}_{\mathbf{i}} \geqslant 0 \tag{8}
\end{equation*}
$$

and $v_{i t}$ is i.i.d. with normal density function,

$$
\begin{equation*}
g(v)=1 / \sqrt{2 \pi \sigma_{v}^{2}} \cdot \exp \left\{-v_{i t} / 2 \sigma_{v}^{2}\right\},-\infty<v_{i t}<+\infty \tag{9}
\end{equation*}
$$

where $u_{i}$ and $v_{i t}$ are independent.
The joint density function for the composed error, $\psi\left(\varepsilon_{i}\right)$ for a specific unit $i$ has the following form:

$$
\begin{equation*}
\psi\left(\varepsilon_{i 1}, \ldots, \varepsilon_{i T}\right)=\int_{0}^{\infty} h(u) \sum_{t=1}^{\mathrm{T}} \mathrm{~g}\left(\varepsilon_{i t}+u\right) d u \tag{10}
\end{equation*}
$$

and the likelihood for all firms (assuming independence) is

$$
\begin{equation*}
\mathrm{L}=\prod_{\mathrm{i}=1}^{\mathrm{N}} \psi\left\{\mathrm{y}_{\mathrm{i} 1}-\mathrm{f}\left(\mathbf{x}_{\mathrm{i} 1}, \boldsymbol{\beta}\right), \ldots, \mathrm{y}_{\mathrm{it}}-\mathrm{f}\left(\mathbf{x}_{\mathrm{iT}}, \boldsymbol{\beta}\right)\right\} \tag{11}
\end{equation*}
$$

We next consider an index of technical inefficiency for a specific firm i represented by the mean value of the difference between the estimated frontier function and observed data over T time periods,

$$
\begin{equation*}
\hat{u}_{i}=1 / T \sum_{i=1}^{T}\left\{f\left(x_{i t}, \hat{\beta}\right)-y_{i t}\right\} \tag{12}
\end{equation*}
$$

where $\hat{\beta}$ is the mle of $\beta{ }^{14}$ Here, $\hat{\mathrm{u}}_{\mathrm{i}}$ is the technial inefficiency of the i -th firm in terms of foregone output.

As with mle, the gls estimator is based on an assumption that technical inefficiency is not correlated with the inputs. However, no specific form for the distribution of technical inefficiency is assumed.

Consider the variance-covariance matrix of the error components:

$$
\begin{equation*}
E\left(\varepsilon \varepsilon^{\prime}\right)=\sigma_{u}^{2}\left(I_{\mathrm{n}} \otimes \mathbf{l}_{\mathrm{T}} \mathbf{l}_{\mathrm{T}}^{\prime}\right)+\sigma_{\mathrm{v}}^{2} \mathbf{I}_{\mathrm{NT}}=\Omega \tag{13}
\end{equation*}
$$

where $1_{T}$ is a vector of l's of order $T \times 1$. Since the variance-covariance matrix is not scalar, direct application of a nonlinear least squares (nlls) method leads to inefficient coefficient estimates and biased variance estiamtes. In order to reduce (13) to a scalar form, we use the transformation given in Fuller and Battese [1973]:

$$
\begin{equation*}
\mathbf{P}=\mathrm{I}_{\mathrm{T}}-\left(1-\sigma_{\mathrm{v}} / \sigma_{1}\right)\left\{\left(1_{\mathrm{T}} \mathrm{l}_{\mathrm{T}}^{\prime}\right) / \mathrm{T}\right\} \tag{14}
\end{equation*}
$$

where $P P^{\prime}=\sigma_{v}^{2} \Omega^{-1}, P_{i} \mathbf{P}_{i}=\sigma_{v}^{2} \Omega_{i}^{-1}$, and $\sigma_{1}^{2}=T \cdot \sigma_{u}^{2}+\sigma_{v}^{2}$.
By applying the transformation $P$ for all NT observations of (7), we get

$$
\begin{equation*}
\left(\mathrm{y}_{\mathrm{it}}-\gamma \overline{\mathrm{y}}_{\mathrm{i}}\right)=(1-\gamma) \mu+\left\{\mathrm{f}\left(\mathrm{x}_{\mathrm{it}}, \beta\right)-\gamma \overline{\mathrm{f}}\left(\mathrm{x}_{\mathrm{it}}, \beta\right)\right\}+\mathrm{v}_{\mathrm{it}} \tag{15}
\end{equation*}
$$

where

$$
\begin{aligned}
& \overline{\mathrm{y}}_{\mathrm{i}}=1 / \mathrm{T} \sum_{\mathrm{t}=1}^{\mathrm{T}} \mathrm{y}_{\mathrm{it}} \\
& \overline{\mathrm{f}}\left(\mathrm{x}_{\mathrm{it}}, \beta\right)=1 / \mathrm{T} \sum_{\mathrm{t}=1}^{\mathrm{T}} \mathrm{f}\left(\mathrm{x}_{\mathrm{it}}, \beta\right),
\end{aligned}
$$

and $\gamma=1-\left(\sigma_{v} / \sigma_{1}\right)$.
After we transform the original model (7), we apply nlls to (15). However, this transformed model (15) depends on $\gamma$ and thus on $\sigma_{\mathrm{v}}^{2}$ and $\sigma_{\mathrm{u}}^{2}$. Consistent estimation of $\sigma_{v}^{2}$ and $\sigma_{u}^{2}$ allows us to construct the feasible GLS estimation [Judge et al. 1985]. The derivation of firm-specific technical inefficiency is based on the same procedure as outlined in (12).

The within estimator utilizes the variation of the variables within the individual firm. As with mle and gls, the firm-specific effect is assumed to be time invariant. However, we need not assume independence of inputs and technical inefficiency. Therefore, the assumptions of the within estimator are more tenable and less restrictive than those of mle and gls.

From (1), we regard ( $-u_{i}$ ) as a dummy variable which is specific to the $i$-th firm. This leads to the incidental parameters problem asymptotically since we should include a dummy variable for each firm. Therefore, we circumvent the problem by using the within transformation [Arora 1973; Judge et al.]. After expressing all data in terms of deviations from individual firm means over time periods we apply nlls. The transformed model is

$$
\begin{equation*}
y_{i t}-\bar{y}_{i}=\left\{f\left(x_{i}, \beta\right)-\bar{f}\left(x_{i}, \beta\right)\right\}+v_{i t}-\bar{v}_{i} \tag{16}
\end{equation*}
$$

where

$$
\begin{aligned}
& \overline{\mathrm{y}}_{\mathrm{i}}=1 / \mathrm{T} \sum_{\mathrm{i}=1}^{\mathrm{T}} \mathrm{y}_{\mathrm{it}}, \\
& \overline{\mathrm{f}}\left(\mathrm{x}_{\mathrm{i}}, \beta\right)=1 / \mathrm{T} \sum_{\mathrm{i}=1}^{\mathrm{T}} \mathrm{f}\left(\mathrm{x}_{\mathrm{it}}, \beta\right) \\
& \overline{\mathrm{v}}_{\mathrm{i}}=1 / \mathrm{T} \sum_{\mathrm{i}=1}^{\mathrm{T}} \mathrm{v}_{\mathrm{it}} .
\end{aligned}
$$

We can represent (16) in a simple form as

$$
\begin{equation*}
w_{i t}=g\left(x_{i t}, \beta\right)+v_{i t}^{*} \tag{17}
\end{equation*}
$$

where

$$
\begin{aligned}
& w_{i t}=y_{i t}-\bar{y}_{i}, v_{i t}{ }^{*}=v_{i t}-\bar{v}_{i} \\
& g\left(x_{i t}, \beta\right)=f\left(x_{i t}, \beta\right)-\bar{f}\left(x_{i t}, \beta\right)
\end{aligned}
$$

By applying nlls to (17), we can derive the within estimator. Estimation of technical inefficiency for an individual firm, $\alpha_{i}$, requires averaging residuals for firm i over the T time periods,

$$
\begin{equation*}
\hat{\alpha}_{\mathrm{i}}=1 / \mathrm{T} \sum_{\mathrm{i}=1}^{\mathrm{T}}\left\{\mathrm{y}_{\mathrm{it}}-\mathrm{f}\left(\mathrm{x}_{\mathrm{it}}, \hat{\beta}\right)\right\} \tag{18}
\end{equation*}
$$

where $\hat{\beta}=$ the within estimator of $\beta$.
For $T \rightarrow \infty$, a consistent estimate of technical efficiency of the most efficient firm is Max $\left(\alpha_{i}\right)$. We can calculate the relative technical efficiency of each firm, $\alpha_{i}^{*}$, as

$$
\begin{align*}
& \alpha_{i}^{*}=\operatorname{Max}\left(\alpha_{i}\right)-\alpha_{i},  \tag{19}\\
& i=1, \ldots, N,
\end{align*}
$$

in terms of the most efficient firm in an industry. Even though the within estimator cannot separate firm-specific technical inefficiency from a common factor which does not vary over time (e.g., capital stock, in a short-run cost function), an es-
timate of this common factor is not necessary to calculate relative technical inefficiency by (19).

## 4. The design of experiments

We consider the CRESH production technology with a single output and three inputs as the true technology. The number of observations is to be decided by the combination of the number of firms ( $\mathrm{N}=10,30,50$ ) and the number of time periods $(T=10,50,90)$. The first step is to generate three inputs. The three inputs are drawn randomly and independently from a lognormal distribution and are fixed through all experiments. Thus the inputs are treated in our experiments as exogenous. We can clearly draw the inputs from many possible distributions, for example, a lognormal distribution [Guilkey and Lovell; Guilkey, Lovell and Sickles] or a uniform distribution [Nerlove 1971; and Arora]. The choice of a lognormal distribution is appropriate when the value of a random variable is regarded as representing the joint effect of a large number of independent variables, so that the effect of a random change is in every case proportional to the previous value of the quantity.

The second step is to decide what values to assign the parameters $\left(\theta, \rho_{i}, \rho, \delta_{i}, \gamma\right)$ which depends on the specific true technology. Since the nature of a true technology may quite profoundly affect the properties of the estimates which are obtained by various methods, it is important to know its exact nature, by choosing its corresponding parameters.

The third step is to generate $\mathbf{N} \cdot \mathrm{T}$ two-error components which consist of $\mathrm{N} \cdot \mathrm{T}$ assumed symmetric and non-symmetric disturbances, respectively. Generally, the $\mathrm{N} \cdot \mathrm{T}$ observations of $\mathrm{v}_{\mathrm{it}}$ are to be randomly and independently drawn from a normal distribution, $\mathrm{N}\left(0, \sigma_{v}^{2}\right)$. The $\mathrm{N} \cdot \mathbf{T}$ observations of $\mathbf{u}_{\mathrm{i}}$ are to be selected from a half-normal distribution for each experiment, based on absolute values from $\mathrm{N}\left(0, \sigma_{\omega}^{2}\right)$ and are constant over time. To characterize the case in which (1) there exists noise in the generated data to confound the measurement of technical inefficiency and (2) technical inefficiency dominates statistical noise, we assume that $\sigma_{u}^{2}$ and $\sigma_{v}^{2}$ are 1.03 and 0.505 , respectively. ${ }^{15}$

Finally, we generate $\mathrm{N} \cdot \mathrm{T}$ output observations by means of

$$
\begin{equation*}
y_{i t} e^{\theta u_{i t}}=\left(\delta_{1} x(1)_{i t}^{-\rho_{1}}+\delta_{2} x(2)_{i t}^{-p_{2}}+\delta_{3} x(3)_{i t}^{-\rho_{3}}\right)^{-\gamma / \rho}+v_{i t}-u_{i} \tag{20}
\end{equation*}
$$

Using the production function (20), we transform technical inefficiency in terms of foregone output (that is, $-u_{i}$ ) into foregone inputs and construct ( $y_{i t}^{*}, x(1)_{i t}^{*}, x(2)_{i t}^{*}$, $x(3)_{i j}^{*}$. Here, $x(k)_{i t}^{*}$ represents the amount of input $k$ including technical inefficiency, and $y_{i t}^{*}$ is the stochastic frontier output. In each replication, we generate a set of data consisting of an output and three inputs.

To use frontier cost functions in estimating technical inefficiency, it is necessary to transform the data base for a frontier production function into that of a frontier cost function. Under the assumption that each firm is a profit maximizer, input prices are generated by means of

$$
\begin{align*}
& w(k)_{i t}=e^{-\theta y_{i t}}\left(1+\theta y_{i t}\right)^{-1}\left(\gamma \delta(k) \rho_{k} / \rho\right) x\left(k_{i t}^{-\left(1+\rho_{k}\right)}\right. \\
& \left(\sum_{k=1}^{3} \delta_{k} x(k)_{i t}^{-(\gamma+\rho) / \rho} e^{\varepsilon_{i t}} .\right. \tag{21}
\end{align*}
$$

where $\varepsilon_{\mathrm{it}} \sim \mathrm{N}\left(0, \sigma_{\varepsilon}^{2}\right)$ is designed to capture random errors in profit maximiation.
Next, the observed cost data including technical inefficiency is obtained by means of

$$
\begin{equation*}
C\left(y^{\theta y}, w(1), w(2), w(3)\right)_{i t}=\sum_{k=1}^{3}\left(w(k)_{i t} x(k)_{i t}^{*}\right) \tag{22}
\end{equation*}
$$

Thus, we have the data base, $\mathrm{C}(\cdot), \mathrm{w}(1), \mathrm{w}(2)$ and $\mathrm{w}(3)$ for the use in a stochastic frontier cost model.

## 5. Experimental results

Pitt and Lee and Schmidt and Sickles introduced models in which firm-specific productive efficiency could be estimated using panel data. These two studies are based on relatively small data sets and an unknown technology. Pitt and Lee use a sample of 50 Indonesian weaving firms, each observed for three years. Schmidt and Sickles use a sample of twelve U.S. airlines, each observed for 35 quarters. Estimation of firm-specific productive efficiency and examination of its finite sample properties would benefit from Monte Carlo techniques.

The main purpose of this section is to investigate the finite sample properties of estimated technical inefficiencies from stochastic frontier models using panel data. To be clear how robust the estimates are, we use three different estimatorsmle, gls and the within estimator.

We first assume a CES production function as a special case of the CRESH production function, ${ }^{16}$

$$
\begin{equation*}
y_{i t}=\left(\delta_{1} x(1)^{-\rho}+\delta_{2} x(2)^{-\rho}+\delta_{3} x(3)^{-\rho}\right)^{-1 / p} \tag{23}
\end{equation*}
$$

where $\delta_{1}, \delta_{2}$ and $\delta_{3}>0 ; \Sigma \delta_{k}=1 ;-1<\rho \neq 0 ; \mathbf{x}(1), \mathbf{x}(2)$ and $\mathbf{x}(3)$ are the three inputs.

We add two error components to (23) and form the stochastic production function:

$$
\begin{equation*}
y_{i t}=\left(\delta_{1} x(1)^{-\rho}+\delta_{2} x(2)^{-\rho}+\delta_{3} x(3)^{-\rho}\right)^{-1 / \rho}-u_{i}+v_{i t} \tag{24}
\end{equation*}
$$

where $u_{i}$ has a non-negative distribution independent of inputs and $v_{i t}$ is assumed to be i.i.d. $N\left(0, \sigma_{v}^{2}\right)$, independent of $u_{i}$ and inputs. Firm-specific technical inefficiency and its finite sample properties are first examined using (24).

The stochastic frontier cost function can also be used to estimate firm-specific inefficiency. One way to derive the stochastic CES cost function is to use a known CES production function and conditions for allocative efficiency. We require the firm to be allocatively efficient by assuming that it operates on its least cost expansion path. Given input prices, the three conditions for allocative efficiency based on the known CES technology are:

$$
\begin{align*}
& \mathrm{w}(1)_{\mathrm{it}} / \mathrm{w}(2)_{\mathrm{it}}=\left(\delta_{1} / \delta_{2}\right) \cdot\left(\mathrm{x}(2)_{\mathrm{it}}^{*} / \mathrm{x}(1)_{\mathrm{it}}^{*}\right)^{1+\rho}  \tag{25}\\
& \mathrm{w}(1)_{\mathrm{it}} / \mathrm{w}(3)_{\mathrm{it}}=\left(\delta_{\mathrm{t}} / \delta_{3}\right) \cdot\left(\mathrm{x}(3)_{\mathrm{it}}^{*} / \mathrm{x}(1)_{\mathrm{it}}^{*}\right)^{1+\rho}  \tag{26}\\
& \mathrm{w}(2)_{\mathrm{it}} / \mathrm{w}(3)_{\mathrm{it}}=\left(\delta_{2} / \delta_{3}\right) \cdot\left(\mathrm{x}(3)_{\mathrm{it}}^{*} / \mathrm{x}(2)_{\mathrm{it}}^{*}\right)^{1+\rho} \tag{27}
\end{align*}
$$

where $w(1), w(2)$ and $w(3)$ are prices of inputs $x(1)^{*}, x(2)^{*}$ and $x(3)^{*}$. We derive the stochastic input demand equations by combining the stochastic CES production function with the three conditions for allocative efficiency,

$$
\begin{align*}
& x(1)_{i t}^{*}=\left(1 / \Delta(1)_{i t}\right) \cdot\left(y_{i t}-v_{i t}+u_{i}\right)  \tag{28}\\
& x(2)_{i t}^{*}=\left(1 / \Delta(2)_{i t}\right) \cdot\left(y_{i t}-v_{i t}+u_{i}\right)  \tag{29}\\
& x(3)_{i t}^{*}=\left(1 / \Delta(3)_{i t}\right) \cdot\left(y_{i t}-v_{i t}+u_{i}\right) \tag{30}
\end{align*}
$$

where

$$
\begin{aligned}
\Delta(1)_{\mathrm{it}}= & \left\{\delta_{1}+\delta_{2}\left(\mathrm{w}(1)_{\mathrm{it}} \cdot \delta_{2} / \mathrm{w}(1)_{\mathrm{it}} \cdot \delta_{\mathrm{t}}\right)^{-(\rho / 1+\rho)}\right. \\
& \left.+\delta_{3}\left(\mathrm{w}(1)_{\mathrm{it}} \cdot \delta_{3} / \mathrm{w}(3)_{\mathrm{it}} \cdot \delta_{1}\right)^{-(\rho / 1+\rho)}\right\}^{-1 / \rho}, \\
\Delta(2)_{\mathrm{it}}= & \left\{\delta_{1}\left(\mathrm{w}(2)_{\mathrm{it}} \cdot \delta_{1} / \mathrm{w}(1)_{\mathrm{it}} \cdot \delta_{2}\right)^{-(\rho / 1+\rho)}+\delta_{2}\right. \\
& \left.+\delta_{3}\left(\mathrm{w}(2)_{\mathrm{it}} \cdot \delta_{3} / \mathrm{w}(3)_{\mathrm{it}} \cdot \delta_{2}\right)^{-(\mathrm{p} / 1+\rho)}\right\}^{-1 / \mathrm{p}}, \\
\Delta(3)_{\mathrm{it}}= & \left\{\delta_{1}\left(\mathrm{w}(1)_{\mathrm{it}} \cdot \delta_{3} / \mathrm{w}(3) \cdot \delta_{1}\right)^{-(\rho / 1+\rho)}+\delta_{3}\right. \\
& \left.+\delta_{2}\left(\mathrm{w}(2)_{\mathrm{it}} \cdot \delta_{3} / \mathrm{w}(3) \cdot \delta_{2}\right)^{-(\rho / 1+\rho)}\right\}^{-1 / \rho}
\end{aligned}
$$

Note that $\mathbf{x}(\mathrm{k})^{*}, \mathrm{k}=1,2,3$, are bounded from below by the stochastic input demand frontiers,

$$
\hat{x}(k)=\left(1 / \Delta(k)_{i t}\right) \cdot\left(y_{i t}-v_{i t}\right)
$$

The excess amount of input demands above these frontiers are due to the technical inefficiency above, and equal $1 / \Delta(k)_{i t} \cdot u_{i}$ for each input $k$, firm i and time $t$.

Finally, the stochastic cost function can be expressed as

$$
\begin{align*}
& \hat{c}(\mathrm{y}, \mathrm{w}(1), \mathrm{w}(2), \mathrm{w}(3))_{\mathrm{it}}=\sum_{\mathrm{k}} \mathrm{w}(\mathrm{k})_{\mathrm{it}} \cdot \mathrm{x}(\mathrm{k})_{\mathrm{it}}^{*} \\
& \quad=\left\{\mathrm{w}(1)_{\mathrm{it}} / \Delta(1)_{\mathrm{it}}+\mathrm{w}(2)_{\mathrm{it}} / \Delta(2)_{\mathrm{it}}+\mathrm{w}(3) / \Delta(3)_{\mathrm{it}}\right\}\left\{\mathrm{y}_{\mathrm{it}}-\mathrm{v}_{\mathrm{it}}+\mathrm{u}_{\mathrm{i}}\right\} \\
& \quad=\alpha_{\mathrm{it}} \cdot\left\{\mathrm{y}_{\mathrm{it}}-\mathrm{v}_{\mathrm{it}}+\mathrm{u}_{\mathrm{i}}\right\} . \tag{31}
\end{align*}
$$

where $\alpha_{i t}=\Sigma_{k} w(k)_{i t} / \Delta(k)_{i t}$ The term $\alpha_{i t} \cdot u_{i}$ represents the amount by which observed cost exceeds the stochastic cost frontier. The advantage of this expression is that it shows the correct relationship between technical inefficiency in the underlying stochastic production function and derived technical inefficiency in the dual stochastic cost function.

Unlike the derived stochastic Cobb-Douglas cost function suggested by Schmidt and Lovell [1979,1980], the derived stochastic CES cost function (31) is a complex nonlinear functional form and is rather empirically intractable. We do not use the directly derived cost function (31) in the study.

Another way to derive the stochastic CES cost function is to use the known CES production function and conditions for cost minization. We derive the deterministic CES cost function from (23) as:

$$
\begin{align*}
c & (y, w(1), w(2), w(3))_{i t} \\
= & y_{i t}\left\{\delta_{1}^{1 / 1-\rho} \cdot\left(1 / w(1)_{i t}\right)^{\rho / 1-\rho}+\delta_{2}^{1 / 1-\rho} \cdot\left(1 / w(2)_{i t}\right)^{\rho / 1-\rho}\right. \\
& \left.+\delta_{3}^{1 / 1-\rho} \cdot\left(1 / w(3)_{i t}\right)^{\rho / 1-\rho}\right\} . \tag{32}
\end{align*}
$$

The stochastic CES cost function is then

$$
\begin{equation*}
\hat{c}(y, w(1), w(2), w(3))_{i t}=c(y, w(1), w(2), w(3))_{i t}+u_{i}-v_{i t} \tag{33}
\end{equation*}
$$

To evaluate how well the stochastic frontier approach estimates the true firmspecific inefficiencies, we need criteria to evaluate relative performance. We use the correlation coefficient and the rank correlation coefficient between the estimated levels and the true levels of inefficiencies. In Table 1, we report the sample correlation coefficients between the true level of firm-specific technical inefficiency and the estimated levels of inefficiency for $\mathrm{N}=10,30,50$ and $T=10,50$, 90 , averaged over 50 replications. We also report the sample rank correlations in parentheses.

Table 1. Correlation Coefficient and Rank Correlation Coefficient ${ }^{\text {a }}$

| N/T |  |  |  |
| :--- | :---: | :---: | :---: |
| MLE | 10 | 50 | 90 |
| 10 | 0.9131 | 0.9615 | 0.9872 |
|  | $(0.6242)$ | $(0.8060)$ | $(0.7818)$ |
| 30 | 0.8860 | 0.9613 | 0.9793 |
|  | $(0.7250)$ | $(0.9132)$ | $(0.8780)$ |
| 50 | 0.9093 | 0.9770 | 0.9841 |
|  | $(0.7446)$ | $(0.9029)$ | $(0.8764)$ |
| GLS |  |  |  |
| 10 | 0.9228 | 0.9687 | 0.9882 |
|  | $(0.5636)$ | $(0.8060)$ | $(0.9030)$ |
| 30 | 0.9051 | 0.9676 | 0.9785 |
|  | $(0.6903)$ | $(0.9399)$ | $(0.8665)$ |
| 50 | 0.8609 | 0.9536 | 0.9791 |
|  | $(0.6720)$ | $(0.9239)$ | $(0.8456)$ |
| Within |  |  |  |
| 10 | 0.9237 | 0.9582 | 0.9844 |
|  | $(0.6848)$ | $(0.8181)$ | $(0.8424)$ |
| 30 | 0.8609 | 0.9596 | 0.9791 |
|  | $(0.6720)$ | $(0.9239)$ | $(0.8456)$ |
| 50 | 0.9072 | 0.9777 | 0.9837 |
|  | $(0.7531)$ | $(0.9084)$ | $(0.8762)$ |

${ }^{\text {a }}$ Each column/row entry is the correlation coefficient of firm-specific technical inefficiency in terms of foregone output between a true firm-specific technical inefficiency and an estimated firm-specific inefficiency for a sample. Each experiment uses $\theta=0$, $\gamma=1.0, \delta_{1}=\delta_{2}=0.3, \sigma_{u}^{2}=1.03, \sigma_{v}^{2}=0.505$. We also use default values of $\rho=-0.67, \mathrm{~N}=30, \mathrm{~T}=50$. Each set of results is based on 50 replications of the experiments.

Three aspects of the results summarized in Table 1 are worthy of comment. First, regardless of sample size and estimator, all sample correlation coefficients are larger than 0.85 and dominate the corresponding sample correlation coefficients of relative rankings which are also relatively large, ranging from 0.56 to 0.92 . As T increases, we find that the sample correlation coefficients approach one for fixed N as we would expect from asymptotic theory. We also note that relatively high correlations are partially attributed to correct information about the underlying true technology. The effect of approximate functional forms on the estimation of inefficiency will be discussed in the next section. Second, the three estimators are similar in estimating firm-specific technical inefficiency. Third, when we com-
pare the correlation coefficients of efficiency levels with those of efficiency rankings, we find that the correlation coefficients of efficiency levels are always larger than those of efficiency rankings for all samples. This is attributable to loss of information, which is expected when efficiency levels are transformed into efficiency rankings.

Table 2 shows the finite sample properties of firm-specific technical inefficiency estimates using mean-square-error (MSE), bias and variance. Several expected patterns are evident. For all estimators and a relatively large number of firms ( $\mathbf{N}=50$ ), bias, variance and MSE fall as Tincreases. For small $N(N=10)$ in Table 3, we do not see any clear pattern in bias, variance and MSE. Our findings from finite samples correspond to the results of asymptotic analysis as follows: Consistent estimates of firm-specific technical inefficiency using panel data need two ideal conditions, $\mathrm{N} \rightarrow \infty$ and $\mathrm{T} \rightarrow \infty$. If we consider the realtive efficiency of estimators, the gls estimator is more (MSE) efficient than mle and the within estimator for all samples. However, the margin in MSE between the three estimators is negliable for large N and T .

In Table 4, we report the sample correlation between the true level of firmspecific technical efficiency and the estimated level of efficiency in terms of excess cost. The sample correlation coefficient of relative rankings is in parenthesis. The results are quite similar to those of Table 1. The stochastic frontier approach using the dual cost function (that is, the CES cost function) also very well estimates firmspecific technical inefficiency from panel data.

We now assume that our information about the underlying technology is incomplete. We use approximating functional forms to model the true technology and inefficiency. The choice of a functional form is important to this study, since any bias that may be introduced by employing an inappropriate functional form will distort efficiency measurement.

General criteria for the choice of a functional form are discussed in Fuss, McFadden, and Mundlak [1978]. The chosen functional form should (1) impose

Table 2. Firm-specific Technical Inefficiency From a CES Production Function $N=50$

| T |  |  |  | 50 |
| :--- | ---: | :--- | :--- | :--- |
| 90 |  |  |  |  |
|  | MSE | 0.4241 | 0.3743 | 0.3726 |
|  | Bias | 0.0388 | 0.0350 | 0.0335 |
|  | Var | 0.4214 | 0.3723 | 0.3715 |
| GLS | MSE | 0.4198 | 0.3721 | 0.3712 |
|  | Bias | 0.0455 | 0.0348 | 0.0342 |
|  | Var | 0.4164 | 0.3703 | 0.3701 |
|  | MSE | 0.4338 | 0.3802 | 0.3794 |
| Within | Bias | 0.0429 | 0.0326 | 0.0320 |
|  | Var | 0.4306 | 0.3785 | 0.3784 |

${ }^{\text {a }}$ MSE, bias and variance are an average of firm-specific inefficiencies across all firms.

Table 3. Firm-specific Technical Inefficiency from a CES Production Function $N=10$

| T | 10 |  | 50 | 90 |
| :--- | ---: | :--- | :--- | :--- |
|  | MSE | 0.3505 | 0.3327 | 0.3860 |
| $M L E$ | Bias | 0.0562 | 0.0261 | 0.1084 |
|  | Var | 0.3461 | 0.3316 | 0.3739 |
|  | MSE | 0.3359 | 0.3077 | 0.3141 |
| $G L S$ | Bias | 0.0539 | 0.0433 | 0.0424 |
|  | Var | 0.3319 | 0.3053 | 0.3119 |
|  | MSE | 0.3822 | 0.3342 | 0.3412 |
| Within | Bias | 0.0401 | 0.0228 | 0.0265 |
|  | Var | 0.3802 | 0.3334 | 0.3403 |

Table 4. Correlation Coefficient and Rank Correlation Coefficient From the Dual CES Cost Functiona

| N/T |  |  |  |
| :--- | :---: | :---: | :---: |
| MLE | 10 | 50 | 90 |
| 10 | 0.9057 | 0.9603 | 0.9774 |
|  | $(0.5757)$ | $(0.7454)$ | $(0.7818)$ |
| 30 | 0.8312 | 0.9178 | 0.9217 |
|  | $(0.6298)$ | $(0.8144)$ | $(0.7757)$ |
| 50 | 0.9014 | 0.9813 | 0.9849 |
|  | $(0.7495)$ | $(0.9189)$ | $(0.8186)$ |
| $G L S$ |  |  |  |
| 10 | 0.9192 | 0.9687 | 0.9880 |
|  | $(0.6242)$ | $(0.8060)$ | $(0.9272)$ |
| 30 | 0.8751 | 0.9662 | 0.9802 |
|  | $(0.6921)$ | $(0.9217)$ | $(0.8782)$ |
| 50 | 0.9017 | 0.9768 | 0.9792 |
|  | $(0.7495)$ | $(0.9073)$ | $(0.8763)$ |
| Within |  |  |  |
| 10 | 0.9166 | 0.9602 | 0.9855 |
|  | $(0.6242)$ | $(0.8060)$ | $(0.8424)$ |
| 30 | 0.9068 | 0.9666 | 0.9773 |
|  | $(0.7606)$ | $(0.9337)$ | $(0.8589)$ |
| 50 | 0.9082 | 0.9787 | 0.9846 |
|  | $(0.7380)$ | $(0.9030)$ | $(0.8906)$ |

[^1]as little structure as possible on the technology, and (2) be empirically tractable, that is, it should be parsimonious in its use of parameters. We use three flexible forms whose duals cannot be represented in closed form but which satisfy the general criteria: CES-translog [Pollak, Sickles, Wales 1984], generalized-Leontief [Diewert 1971,1973] and translog [Christensen, Jorgensen and Lau 1971,1973], Sargen [1971].

The first approximation we consider is the translog (TL),

$$
\begin{align*}
& \ln c(y, w)=\alpha_{0}+\alpha_{y} \ln y+\frac{1}{2} \cdot \alpha_{y y}(\ln y)^{2} \\
& +\sum_{i} a_{i} \ln w_{i}+\frac{1}{2} \cdot \sum_{i} \sum_{j} a_{i j}\left(\ln w_{i}\right)\left(\ln w_{j}\right) \\
& +\sum_{i} a_{y i}(\ln y)\left(\ln w_{i}\right) ;  \tag{34}\\
& ; \alpha_{i j}=\alpha_{j i} \text { for all } i \neq j .
\end{align*}
$$

Necessary and sufficient conditions insuring that $c(y, w)$ is linearly homogeneous in input prices are:

$$
\begin{equation*}
\sum_{i} \alpha_{i}=1 ; \sum_{j} \alpha_{i j}=0, i=1, \ldots m ; \sum_{i} \alpha_{y i}=0 \tag{35}
\end{equation*}
$$

It is well known that the estimated translog cost function frequently fails to satisfy the concavity condition over the whole range of the input prices that wellbehaved cost functions should possess. Jorgenson and Fraumeni [1981], Jorgenson [1984] and Diewert and Wales [1987], among others, impose global curvature conditions over the set of prices generating nonnegative input demands. A potentially serious problem with these suggestions, however, is that they compromise the flexibility of the translog. Thus we maintain the concavity condition as a testable hypothesis.

Using Hotelling's lemma, a typical cost-minimizing share equation for input $i$ is

$$
\begin{align*}
\mathrm{m}_{\mathrm{i}}(\mathrm{y}, \mathrm{w}) & =\partial \ln \mathrm{c} / \partial \ln w_{\mathrm{i}}=\partial \mathrm{c} \cdot \mathrm{w}_{\mathrm{i}} / \partial \mathrm{w}_{\mathrm{i}} \cdot \mathrm{c}=\mathrm{x}_{\mathrm{i}} \cdot \mathrm{w}_{\mathrm{i}} / \mathrm{c} \\
& =\alpha_{\mathrm{i}}+\sum{ }_{j} \alpha_{\mathrm{ij}}\left(\ln w_{j}\right)+\alpha_{\mathrm{yi}} \ln \mathrm{y}, \mathrm{i}=1, \ldots, \mathrm{~m} . \tag{36}
\end{align*}
$$

As economic measures of the underlying technology, we use the AES, $\sigma_{\mathrm{ij}}(\mathbf{x})$ and returns to scale, $\gamma(\mathrm{x})$ given by

$$
\begin{aligned}
& \gamma(y, w)=\left(\alpha_{y}+\alpha_{y y} \ln y+\sum \alpha_{y \mathrm{i}} \ln w_{i}\right)^{-1}, \\
& \sigma_{\mathrm{ij}}(\mathrm{y}, \mathrm{w})=\left(\alpha_{\mathrm{ij}}+\mathrm{m}_{\mathrm{i}} \cdot \mathrm{~m}_{\mathrm{j}}\right) / \mathrm{m}_{\mathrm{i}} \cdot \mathrm{~m}_{\mathrm{j}}, \text { for } \mathrm{i} \neq \mathrm{j}, \\
& \sigma_{\mathrm{ii}}(\mathrm{y}, \mathrm{w})=\left(\alpha_{\mathrm{ii}}-\mathrm{m}_{\mathrm{i}}+\mathrm{m}_{\mathrm{i}}^{2}\right) /\left(\mathrm{m}_{\mathrm{i}}\right)^{2} .
\end{aligned}
$$

Next, we consider the CES-translog [Pollak, Sickles and Wales], which combines the CES and the translog and thus is a hybrid form. One advantage of the CES-translog is that it is compatible with a wider range of substitution possibilities than either the CES or the translog. The m-input CES-translog is given by

$$
\begin{align*}
\ln c(y, w)= & \alpha_{0}+\alpha_{y} \ln y+\frac{1}{2} \cdot \alpha_{y y}(\ln y)^{2} \\
& +\ln \left[\sum_{\mathrm{j}} \mathrm{a}_{\mathrm{j}} \mathbf{w}_{\mathrm{j}}^{1-\sigma}\right]^{1 /(1-\sigma)} \\
& +\frac{1}{2} \cdot \sum_{\mathrm{j}} \sum_{\mathrm{i}} \mathrm{a}_{\mathrm{ji}} \ln \mathbf{w}_{\mathrm{j}} \ln w_{\mathrm{i}} \\
& +\sum_{\mathrm{i}} \mathrm{a}_{\mathrm{yi}} \ln \mathrm{y} \cdot \ln w_{\mathrm{i}} \tag{37}
\end{align*}
$$

where $\alpha_{i j}=\alpha_{i j}$, for all $\mathrm{i}, \mathrm{j} ; \boldsymbol{\Sigma}_{\mathrm{j}} \alpha_{\mathrm{ij}}=0$, for $\mathrm{i}=1, \ldots, \mathrm{~m} ; \Sigma_{\mathrm{j}} \alpha_{\mathrm{j}}=1 ; \Sigma_{\mathrm{j}} \alpha_{\mathrm{yj}}=0$.
By Shephard's lemma, the cost-minimizing input demand functions, in share form, are given by

$$
\begin{equation*}
m_{i}(y, w)=\left(\alpha_{i} \cdot w_{i}^{1-\sigma}\right) /\left(\sum_{j} \alpha_{j} w_{j}^{1-\sigma}\right)+\sum_{j} \beta_{\mathrm{ij}} \ln w_{j}+\alpha_{y \mathrm{i}} \ln y \tag{38}
\end{equation*}
$$

AES and returns to scale are given by

$$
\begin{align*}
& \gamma(y, w)=\left(\alpha_{y}+\alpha_{y y} \ln y+\sum_{i} \alpha_{y i} \ln w_{i}\right)^{-1} \\
& \alpha_{i j}(y, w)=\left\{(\sigma-1) \cdot\left[\left(\alpha_{i} w_{i}^{1-\sigma}\right) /\left(\sum_{k} \alpha_{k} w_{k}^{1-\sigma}\right)\right]\right. \\
&\left.\cdot\left[\left(\alpha_{j} w_{j}^{1-\sigma}\right) /\left(\sum_{k} \alpha_{k} w_{k}^{1-\sigma}\right)^{2}\right]+\alpha_{i j}+m_{i} m_{j}\right\} / m_{i} m_{j}, \text { for } i \neq j, \\
& \sigma_{i i}(y, w)=\{ \left\{(1-\sigma) \alpha_{i} w_{i}^{1-\sigma} \cdot\left(\sum_{k} \alpha_{k} w_{k}^{1-\sigma}\right)-(1-\sigma) \cdot\left(\alpha_{i} w_{i}^{1-\sigma}\right)^{2}\right] / \\
&\left.\left(\sum_{k} \alpha_{k} w_{k}^{1-\sigma}\right)^{2}+\alpha_{i i}+m_{i}^{2}-m_{i}\right\} / m_{i}^{2} . \tag{39}
\end{align*}
$$

If $\alpha_{i j}=\alpha_{\mathrm{ij}}=0$ for all $\mathrm{i}, \mathrm{j}$, the CES-translog input demand system and cost function reduce to those of the CES. When $\sigma=1$, the CEStranslog system reduces to that of the translog, and as $\sigma$ approaches one, its cost function approaches the translog's.

The final approximation we consider is the generalized-Leontief (GL) which we write as

$$
\begin{equation*}
c(y, w)=y \sum_{i} \sum_{j} \alpha_{i j} w_{i}^{1 / 2} w_{j}^{1 / 2}+y^{2} \sum_{i} \alpha_{i} w_{i}+\sum_{i} \beta_{i} w_{i} \tag{40}
\end{equation*}
$$

where $\alpha_{\mathrm{ij}}=\alpha_{\mathrm{ji}}$ for all $\mathrm{i} \neq \mathrm{j}$. Applying Shephard's lemma, we obtain the system of derived input demand functions as

$$
\begin{equation*}
x_{i}(y, w)=y \sum_{i} \alpha_{i j}\left(w_{j} / w_{i}\right)^{1 / 2}+y^{2} \alpha_{i}+\beta_{i}, \text { for } i=1, \ldots, m \tag{41}
\end{equation*}
$$

Returns to scale and AES are given by

$$
\begin{align*}
& \gamma(x)=\left(\sum_{i} \sum_{j} \alpha_{i j} w_{i}^{1 / 2} w_{j}^{1 / 2}+2 y \sum_{i} \alpha_{i} w_{i}\right)^{-1} \cdot(c(y, w) / y) \\
& \sigma_{i j}(x)=\left(\frac{1}{2} \alpha_{i j} y w_{i}^{-1 / 2} w_{j}^{-1 / 2}\right)\left(c(y, w) / x_{i} x_{j}\right), \text { for } i \neq j \\
& \sigma_{i i}(x)=\left(\frac{1}{2} y w_{i}^{-3} c(y, w) \sum_{j} \alpha_{i j} w_{j}\right) / x_{i}^{2} . \tag{42}
\end{align*}
$$

The GL approximation is non-homothetic unless $\alpha_{i}=\beta_{i}=0$, for all i , in which case it is linearly homogeneous. Thus the GL approximation is incapable of distinguishing among homotheticity, homogeneity and linear homogeneity. If $\alpha_{i j}=0$ for all $\mathrm{i} \neq \mathrm{j}$, the GL approximation collapses to a fixed proportions form.

Using equations (34), (37) and (40), we estimate stochastic frontier models subject to the restrictions implied by symmetry and linear homogeneity. We also estimate the cost function system which consists of a deterministic cost function and, either the share equations in the case of the TL and the CES-TL (see equations (36) and (38)), or the demand equations in the case of the GL (see equation (41)). We impose a simple stochastic structure on the systems: Additive normal errors which are independent over observations and which have a constant covariance matrix. System estimates are based on maximizing the concentrated log likelihood function.

Our findings are summarized in Tables 5-9, each of which depend on the complexity and structure of the underlying technology. We characterize the technology by two factors. The first is the matrix of Allen-Uzawa partial elasticities of substitution among inputs, $\sigma_{\mathrm{ij}}(x)$. We refer to a simple technology as one in which the off-diagonals of the AES matrix are the same, and are bounded away from unity, with input substitution being either easy $\left(\sigma_{i j}(x)>1.33\right)$, or difficult $(0<$ $\sigma_{\mathrm{ij}}(\mathrm{x})<0.303$ ). We consider a complex technology to be one in which a mix of substitution possibilities exists, with substitution being much easier for some input pairs than for others. The second factor is returns to scale, $\gamma(\mathbf{x})$. We look at homogeneous, almost homogeneous, homothetic and almost homothetic technologies. A complete set of technologies is based on permutations of $\sigma_{i j}(x)$ and $\gamma(\mathrm{x})$.

Tables 5-8 report the same statistics as Tables 1,3 and 4, that is, correlation coefficients of firm-specific technical efficiency and its relative ranking. Table 9 tabulates the AES and returns to scale using equations (3), (4) and (5) and their estimates. For each observation in each replication (50) of an experiment, we calculate six AES and returns to scale for total number of observations 1500 ( $=$ Number of firms (30) $\times$ Number of time periods (50)). We report four statistics-median, median deviation, mean and absolute bias for the four estimators. Table 10 reports the set of eigen values for the AES which satisfy the concavity condition among 50 replications. Fractions in parentheses indicate the proportion of times that concavity is satisfied. We now turn to the results of these experiments. There are three main findings.

Table 5. Correlation Coefficient In a Simple Technology

| Experiment |  | CES-translog | Translog | Generalized-Leontief |
| :--- | :--- | :--- | :---: | ---: |
| $1 . \rho=-0.67$ | MLE | 0.8919 | 0.7647 | 0.4292 |
|  | GLS | 0.8912 | 0.8669 | 0.3782 |
|  | Within | 0.8902 | 0.8847 | 0.3793 |
|  | System | 0.4267 | 0.3781 | 0.5376 |
| $2 . \rho=-0.5$ | MLE | 0.9320 | 0.9488 | 0.5775 |
|  | GLS | 0.9320 | 0.9447 | 0.4577 |
|  | Within | 0.9322 | 0.9477 | 0.4651 |
|  | System | 0.4196 | 0.6155 | 0.4962 |
| 3. $\rho=-0.25$ | MLE | 0.9844 | 0.9919 | 0.7146 |
|  | GLS | 0.9841 | 0.9917 | 0.3782 |
|  | Within | 0.9841 | 0.9917 | 0.9219 |
|  | System | 0.7530 | 0.8963 | -0.4384 |
| 4. $\rho=+0.1$ | MLE | 0.2312 | -0.1575 | 0.1931 |
|  | GLS | 0.1277 | 0.1814 | 0.2517 |
|  | Within | 0.0331 | 0.0213 | 0.2562 |
|  | System | 0.1301 | -0.0334 | -0.3353 |
| 5. $\rho=+2.0$ | MLE | 0.6788 | -0.1048 | 0.3848 |
|  | GLS | 0.6794 | -0.1027 | 0.3782 |
|  | Within | 0.6734 | -0.0959 | 0.1193 |
|  | System | -0.0834 | -0.1179 | -0.4048 |

Table 6. Rank Correlation Coefficient In a Simple Technology

| Experiment |  | CES-translog | Translog | Generalized-Leontief |
| :--- | :--- | :--- | ---: | :--- |
| $1 . \rho=-0.67$ | MLE | 0.7721 | 0.7441 | 0.4095 |
|  | GLS | 0.7721 | 0.8358 | 0.4255 |
|  | Within | 0.7721 | 0.8660 | 0.4313 |
|  | System | 0.2391 | 0.1692 | 0.2275 |
| $2 . \rho=-0.5$ | MLE | 0.7272 | 0.8963 | 0.4220 |
|  | GLS | 0.7276 | 0.9047 | 0.4389 |
|  | Within | 0.7268 | 0.8976 | 0.4531 |
|  | System | 0.1190 | 0.2013 | 0.0131 |
| 3. $\rho=-0.25$ | MLE | 0.8478 | 0.9239 | 0.5136 |
|  | GLS | 0.8513 | 0.9274 | 0.4255 |
|  | Within | 0.8473 | 0.9274 | 0.7677 |
|  | System | 0.4513 | 0.1123 | -0.2836 |
| 4. $\rho=+0.1$ | MLE | -0.2341 | -0.0180 | -0.0144 |
|  | GLS | -0.1937 | 0.2627 | -0.0447 |
|  | Within | -0.1474 | 0.1862 | -0.0234 |
|  | System | 0.1296 | -0.0620 | -0.3971 |
| 5. $\rho=+2.0$ | MLE | 0.5644 | 0.0117 | 0.0749 |
|  | GLS | 0.5635 | 0.0629 | 0.4255 |
|  | Within | 0.5639 | 0.0656 | -0.0327 |
|  | System | -0.0033 | -0.0291 | -0.1114 |
|  |  |  |  |  |

1. In Tables 5-6, we vary the input substitution in the underlying CES technology from $\sigma=3.030(\rho=-0.67)$ to $\sigma=0.333(\rho=+2.000)$. Overall, the CES-TL performs well through all possibilities of input substitution. As the underlying technology approaches the Cobb-Douglas, however, the TL dominates by a small margin, although the performance of the TL markedly deteriorates when the true elasticity of sustitution is quite small (a case close to a fixed coefficient technology). The performance of the CES-TL is better than the TL over a wide range of technologies. When we compare the performance of the GL to those of the CESTL and the TL, we find that the GL does a relatively poor job of tacking the level of firm inefficiency through all possibilities of underlying technologies. In addition, no functional form measures technical inefficiency well as the underlying technology approaches fixed coefficients. These results have some intuitive appeal. The Cobb-Douglas is the only CES form compatible with the translog. If we consider that the CES-TL provides a transparent generalization of the CES and the TL, the performance of the CES-TL is better than that of the TL except for a special case of the underlying technology-Cobb-Douglas. We could conject that the GL is better than the two other forms in the case of a fixed coefficient technology. However, we can find no dominance of the GL in this case.

A comparison between the single equation and the systems estimator reveals the systems estimator to be quite poor in estimating firm-specific inefficiency. One justification is that the systems estimator for a flexible functional form models the error structure in a very ad hoc fashion. As Bauer et al. have pointed out, researchers whose primary goal is the study of industry technology estimate the system of cost and share equations (ignoring any inefficiency), whereas researchers of inefficiency estimate only the cost (production) function (foregoing the additional information embodied in the share equations).

Tables 7 and 8 point out that as the complexity of the underlying technology increases, the performance of the stochastic frontier model markedly deteriorates. One reason is that in a simple technology shortfalls from the frontier are assumed to be the result of technical inefficiencies. However, in a complex technology, we conject that the characteristics of the underlying technology, such as returns to scale or the AES, are different at the frontier than away from the frontier [Forsund and Jansen 1977; Førsund and Hjamarsson 1979a,b]. Thus, we need to find a reasonable specification which allows one to examine features of the technology at varying distances from the frontier [Schmidt 1985]. We still do not yet have a functional form with this characteristic. If the underlying technology has this potential characteristic, we can use non-parametric methods, avoiding the danger of distorting the efficiency measurement by imposing an incorrect parametric form.

Based on Tables 5-8, we conclude that the ability of the stochastic frontier models to estimate firm-level inefficiency is quite sensitive to the complexity and structure of the underlying technology.
2. Tables 1-6 indicate that the correlation coefficients of firm-specific technical inefficiency and its relative ranking are very similar for the three estimators (mle,

Table 7. Correlation Coefficient In a Complex Technology

| Experiment |  | CES-translog | Translog | Generalized-Leontief |
| :--- | :--- | :---: | :---: | :---: |
| 6. $\rho=-0.45$ | MLE | 0.7957 | 0.0417 | 0.1512 |
| $\rho_{1}=-0.4$ | GLS | 0.0559 | 0.0544 | 0.0213 |
| $\rho_{2}=-0.5$ | Within | 0.0537 | 0.0544 | 0.0212 |
| $\rho_{3}=-0.6$ | System | 0.3929 | 0.1033 | 0.1083 |
| 7. $\rho=-0.85$ | MLE | 0.5133 | 0.3192 | 0.2262 |
| $\rho_{1}=-0.7$ | GLS | 0.2636 | 0.3414 | 0.1466 |
| $\rho_{2}=-0.8$ | Within | 0.2622 | 0.3419 | 0.1466 |
| $\rho_{3}=-0.9$ | System | 0.2369 | 0.2033 | 0.1083 |

Table 8. Rank Correlation Coefficient In a Complex Technology

| Experiment |  | CES-translog | Translog | Generalized-Leontief |
| :--- | :--- | :---: | :---: | :---: |
| 6. $\rho=-0.45$ | MLE | 0.5590 | 0.4672 | 0.2910 |
| $\rho_{1}=-0.4$ | GLS | 0.5189 | 0.5002 | 0.2062 |
| $\rho_{2}=-0.5$ | Within | 0.5171 | 0.5043 | 0.2091 |
| $\rho_{3}=-0.6$ | System | 0.3212 | 0.4027 | 0.4534 |
| 7. $\rho=-0.85$ | MLE | 0.3828 | 0.3486 | 0.2516 |
| $\rho_{1}=-0.7$ | GLS | 0.3713 | 0.3539 | 0.3014 |
| $\rho_{2}=-0.8$ | Within | 0.3730 | 0.3472 | 0.3014 |
| $\rho_{3}=-0.9$ | System | 0.3025 | 0.3121 | 0.5385 |

gls and the within estimator). Since the within estimator is obtained without two strong assumptions, it would tend to be preferred.
3. Flexible functional form estimates frequently fail to satisfy the appropriate theoretical curvature conditions over finite regions (rather than at a single point). Wales [1977], Caves and Christensen [1980] and Barnett and Lee [1985] compared various flexible functional forms with respect to their regions in a parameter space where curvature conditions are satisfied. Since $\left[\sigma_{i j}\right], i, j=1,2,3$ is a quadratic form of the cost function's Hessian, we can test the concavity of flexible forms using [ $\sigma_{\mathrm{ij}}$ ]. For the TL and the GL, we have results quite similar to those of Caves and Christensen. The GL has good regional properties when substitutability is very low and the TL has good regional properties when all elasticities of substitution are near one. These results are not surprising, since the GL is known to satisfy theoretical conditions everywhere in the Leontief technology; and the TL is known to satisfy theoretical regularity conditions everywhere if the true technology is the Cobb-Douglas. In the comparison among three functional forms, the CES-TL dominates two other forms over a wide range of technology. Thus, we prefer the CES-TL to the TL or the GL with respect to the satisfaction of concavity condition.

Table 9.1. CES-translog

1. $\rho=-0.67$

|  | $\sigma_{12}$ | $\sigma_{13}$ | $\sigma_{23}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{33}$ | $\gamma$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| True | 3.030 | 3.030 | 3.030 | -7.861 | -7.538 | -4.977 | 1.000 |
| MLE | 2.929 | 2.636 | 2.614 | -6.736 | -6.358 | -3.897 | 1.041 |
|  | $(0.101)$ | $(0.394)$ | $(0.416)$ | $(1.125)$ | $(1.179)$ | $(1.080)$ | $(0.041)$ |
|  | 2.937 | 2.633 | 2.621 | -6.737 | -6.371 | -3.917 | 1.042 |
|  | $(0.092)$ | $(0.396)$ | $(0.408)$ | $(1.124)$ | $(1.167)$ | $(1.059)$ | $(0.042)$ |
| GLS | 2.921 | 2.644 | 2.619 | -6.720 | -6.343 | -3.905 | 1.042 |
|  | $(0.108)$ | $(0.385)$ | $(0.410)$ | $(1.141)$ | $(1.182)$ | $(1.068)$ | $(0.042)$ |
|  | 2.932 | 2.639 | 2.623 | -6.710 | -6.343 | -3.905 | 1.042 |
|  | $(0.097)$ | $(0.381)$ | $(0.407)$ | $(1.150)$ | $(1.195)$ | $(1.072)$ | $(0.042)$ |
| Within | 2.931 | 2.637 | 2.612 | -6.738 | -6.361 | -3.900 | 1.041 |
|  | $(0.098)$ | $(0.393)$ | $(0.417)$ | $(1.123)$ | $(1.176)$ | $(1.076)$ | $(0.041)$ |
|  | 2.940 | 2.631 | 2.616 | -6.727 | -6.353 | -3.899 | 1.042 |
|  | $(0.089)$ | $(0.398)$ | $(0.413)$ | $(1.134)$ | $(1.185)$ | $(1.078)$ | $(0.042)$ |
| System | 3.017 | 2.556 | 2.589 | -6.954 | -6.686 | -3.750 | 0.817 |
|  | $(0.019)$ | $(0.473)$ | $(0.440)$ | $(0.906)$ | $(0.851)$ | $(1.227)$ | $(0.182)$ |
|  | 3.017 | 2.556 | 2.589 | -6.959 | -6.684 | -3.752 | 0.817 |
|  | $(0.013)$ | $(0.473)$ | $(0.445)$ | $(0.902)$ | $(0.853)$ | $(1.225)$ | $(0.182)$ |
|  |  |  |  |  |  |  |  |

Table 9.2. Translog
2. $\rho=-0.67$

|  | $\sigma_{12}$ | $\sigma_{13}$ | $\sigma_{23}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{33}$ | $\gamma$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| True | 3.030 | 3.030 | 3.030 | -7.861 | -7.538 | -4.977 | 1.000 |
| MLE | 2.760 | 3.056 | 2.829 | -6.407 | -5.711 | -4.202 | 1.043 |
|  | $(0.270)$ | $(0.045)$ | $(0.200)$ | $(1.454)$ | $(1.826)$ | $(0.775)$ | $(0.043)$ |
|  | 2.699 | 3.017 | 2.859 | -6.231 | -5.876 | -4.191 | 1.070 |
|  | $(0.330)$ | $(0.012)$ | $(0.171)$ | $(1.630)$ | $(1.662)$ | $(0.786)$ | $(0.070)$ |
| GLS | 2.757 | 3.054 | 2.814 | -6.483 | -5.731 | -4.148 | 1.044 |
|  | $(0.272)$ | $(0.032)$ | $(0.216)$ | $(1.378)$ | $(1.807)$ | $(0.829)$ | $(0.044)$ |
|  | 2.751 | 3.056 | 2.813 | -6.474 | -5.726 | -4.154 | 1.045 |
|  | $(0.278)$ | $(0.026)$ | $(0.216)$ | $(1.387)$ | $(1.811)$ | $(0.823)$ | $(0.045)$ |
| Within | 2.773 | 3.040 | 2.833 | -6.401 | -5.736 | -4.214 | 1.043 |
|  | $(0.256)$ | $(0.035)$ | $(0.196)$ | $(1.460)$ | $(1.801)$ | $(0.763)$ | $(0.043)$ |
|  | 2.775 | 3.040 | 2.831 | -6.406 | -5.725 | -4.221 | 1.044 |
|  | $(0.255)$ | $(0.010)$ | $(0.199)$ | $(1.455)$ | $(1.812)$ | $(0.755)$ | $(0.044)$ |
| System | 2.785 | 2.900 | 2.873 | -6.218 | -5.825 | -4.156 | 1.005 |
|  | $(0.244)$ | $(0.130)$ | $(0.156)$ | $(1.643)$ | $(1.712)$ | $(0.821)$ | $(0.005)$ |
|  | 2.798 | 2.892 | 2.874 | -6.222 | -5.844 | -4.159 | 1.006 |
|  | $(0.231)$ | $(0.138)$ | $(0.1559)$ | $(1.639)$ | $(1.693)$ | $(0.818)$ | $(0.006)$ |
|  |  |  |  |  |  |  |  |

Table 9.3. Generalized-Leontief
2. $\rho=-0.67$

|  | $\sigma_{12}$ | $\sigma_{13}$ | $\sigma_{23}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{33}$ | $\gamma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| True | 3.030 | 3.030 | 3.030 | -7.861 | -7.538 | -4.977 | 1.000 |
| MLE | 1.454 | 2.910 | 2.895 | -1.601 | -1.573 | -1.296 | 1.030 |
|  | (1.575) | (0.122) | (0.134) | (6.260) | (5.965) | (3.681) | (0.030) |
|  | 1.457 | 2.931 | 2.908 | -1.601 | -1.579 | -1.301 | 1.033 |
|  | (1.573) | (0.098) | (0.121) | (6.260) | (5.958) | (3.676) | (0.033) |
| GLS | 1.459 | 2.906 | 2.887 | -1.604 | -1.570 | -1.303 | 1.032 |
|  | (1.570) | (0.124) | (0.142) | (6.257) | (5.967) | (3.674) | (0.032) |
|  | 1.462 | 2.981 | 2.887 | -1.604 | -1.569 | -1.303 | 1.032 |
|  | (1.567) | (0.118) | (0.143) | (6.257) | (5.968) | (3.674) | (0.032) |
| Within | 1.457 | 2.910 | 2.890 | -1.604 | -1.569 | -1.303 | 1.032 |
|  | (1.573) | (0.119) | (0.139) | (6.257) | (5.969) | (3.674) | (0.032) |
|  | 1.454 | 2.929 | 2.892 | -1.604 | -1.567 | -1.303 | 1.031 |
|  | (1.575) | (0.110) | (0.037) | (9.695) | (9.481) | (5.620) | (0.031) |
| System | 1.830 | 2.883 | 2.958 | -1.481 | -1.420 | -1.225 | 0.982 |
|  | (1.199) | (0.146) | (0.716) | (6.380) | (6.118) | (3.752) | (0.017) |
|  | 1.832 | 2.884 | 2.960 | -1.481 | -1.41 | -1.224 | 0.982 |
|  | (1.198) | (0.146) | (0.069) | (6.380) | (6.118) | (3.753) | (0.017) |

Table 9.4. CES-translog

1. $\rho=-0.25$

|  | $\sigma_{12}$ | $\sigma_{13}$ | $\sigma_{23}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{33}$ | $\gamma$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| True | 1.333 | 1.333 | 1.333 | -3.189 | -3.116 | -2.018 | 1.000 |
| MLE | 0.682 | 0.940 | 0.971 | -1.583 | -1.615 | -2.128 | 1.650 |
|  | $(0.666)$ | $(0.385)$ | $(0.355)$ | $(1.619)$ | $(1.529)$ | $(0.112)$ | $(0.643)$ |
|  | 0.666 | 0.947 | 0.977 | -1.570 | -1.587 | -2.126 | 1.643 |
|  | $(0.650)$ | $(0.392)$ | $(0.362)$ | $(1.606)$ | $(1.501)$ | $(0.110)$ | $(0.650)$ |
| GLS | 0.662 | 0.947 | 0.983 | -1.532 | -1.578 | -2.106 | 1.647 |
|  | $(0.670)$ | $(0.385)$ | $(0.350)$ | $(1.656)$ | $(1.538)$ | $(0.088)$ | $(0.647)$ |
|  | 0.655 | 0.946 | 0.983 | -1.531 | -1.587 | -2.102 | 1.659 |
|  | $(0.677)$ | $(0.386)$ | $(0.349)$ | $(1.657)$ | $(1.529)$ | $(0.084)$ | $(0.659)$ |
| Within | 0.666 | 0.947 | 0.983 | -1.532 | -1.573 | -2.106 | 1.647 |
|  | $(0.666)$ | $(0.385)$ | $(0.350)$ | $(1.656)$ | $(1.543)$ | $(0.088)$ | $(0.647)$ |
|  | 0.654 | 0.947 | 0.983 | -1.531 | -1.586 | -2.103 | 1.660 |
|  | $(0.678)$ | $(0.385)$ | $(0.349)$ | $(1.658)$ | $(1.530)$ | $(0.085)$ | $(0.660)$ |
| System | 0.656 | 0.890 | 0.927 | -1.499 | -1.563 | -1.955 | 0.756 |
|  | $(0.677)$ | $(0.442)$ | $(0.405)$ | $(1.690)$ | $(1.553)$ | $(0.062)$ | $(0.243)$ |
|  | 0.661 | 0.891 | 0.928 | -1.505 | -1.568 | -1.964 | 0.755 |
|  | $(0.672)$ | $(0.441)$ | $(0.404)$ | $(1.684)$ | $(1.548)$ | $(0.053)$ | $(0.244)$ |
|  |  |  |  |  |  |  |  |

Table 9.5. Translog
2. $\rho=-0.25$

|  | $\sigma_{12}$ | $\sigma_{13}$ | $\sigma_{23}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{33}$ | $\gamma$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| True | 1.333 | 1.333 | 1.333 | -3.189 | -3.116 | -2.018 | 1.000 |
| MLE | 0.650 | 1.007 | 0.972 | -1.590 | -1.551 | -2.143 | 1.656 |
|  | $(0.683)$ | $(0.325)$ | $(0.361)$ | $(1.599)$ | $(1.565)$ | $(0.127)$ | $(0.656)$ |
|  | 0.647 | 1.007 | 0.969 | -1.584 | -1.553 | -2.132 | 1.662 |
|  | $(0.686)$ | $(0.325)$ | $(0.363)$ | $(1.604)$ | $(1.563)$ | $(0.114)$ | $(0.662)$ |
| GLS | 0.646 | 1.009 | 0.974 | -1.591 | -1.552 | -2.146 | 1.661 |
|  | $(0.689)$ | $(0.317)$ | $(0.358)$ | $(1.598)$ | $(1.559)$ | $(0.121)$ | $(0.676)$ |
|  | 0.643 | 1.015 | 0.974 | -1.591 | -1.557 | -2.139 | 1.676 |
|  | $(0.686)$ | $(0.323)$ | $(0.359)$ | $(1.597)$ | $(1.564)$ | $(0.128)$ | $(0.661)$ |
| Within | 0.648 | 1.010 | 0.973 | -1.592 | -1.552 | -2.146 | 1.663 |
|  | $(0.684)$ | $(0.322)$ | $(0.359)$ | $(1.596)$ | $(1.564)$ | $(0.128)$ | $(0.663)$ |
|  | 0.643 | 1.015 | 0.974 | -1.589 | -1.556 | -2.139 | 1.678 |
|  | $(0.689)$ | $(0.318)$ | $(0.359)$ | $(1.599)$ | $(1.560)$ | $(0.121)$ | $(0.678)$ |
| System | 0.662 | 0.911 | 0.939 | -1.496 | -1.532 | -2.024 | 1.233 |
|  | $(0.670)$ | $(0.422)$ | $(0.394)$ | $(1.692)$ | $(1.584)$ | $(0.017)$ | $(0.233)$ |
|  | 0.663 | 0.905 | 0.937 | -1.489 | -1.533 | -2.025 | 1.232 |
|  | $(0.669)$ | $(0.427)$ | $(0.396)$ | $(1.699)$ | $(1.583)$ | $(0.007)$ | $(0.237)$ |
|  |  |  |  |  |  |  |  |

Table 9.6. Generalized-Leontief
2. $\rho=-0.25$

|  | $\sigma_{12}$ | $\sigma_{13}$ | $\sigma_{23}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{33}$ | $\gamma$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| True | 1.333 | 1.333 | 1.333 | -3.189 | -3.116 | -2.018 | 1.000 |
| MLE | 0.452 | 1.417 | 0.983 | -1.978 | -1.960 | -1.907 | 1.292 |
|  | $(0.881)$ | $(0.184)$ | $(0.350)$ | $(1.211)$ | $(1.156)$ | $(0.118)$ | $(0.320)$ |
|  | 0.392 | 1.352 | 0.885 | -1.757 | -1.758 | -1.723 | 1.147 |
|  | $(0.940)$ | $(0.019)$ | $(0.448)$ | $(1.432)$ | $(1.358)$ | $(0.294)$ | $(0.147)$ |
| GLS | 1.459 | 2.906 | 2.887 | -1.604 | -1.570 | -1.303 | 1.032 |
|  | $(1.570)$ | $(0.124)$ | $(0.142)$ | $(6.257)$ | $(5.967)$ | $(3.674)$ | $(0.032)$ |
|  | 1.462 | 2.912 | 2.887 | -1.604 | -1.569 | -1.303 | 1.032 |
|  | $(1.567)$ | $(0.118)$ | $(0.143)$ | $(6.257)$ | $(5.968)$ | $(3.674)$ | $(0.032)$ |
| Within | 0.403 | 1.289 | 0.926 | -1.555 | -1.623 | -1.639 | 1.059 |
|  | $(0.930)$ | $(0.098)$ | $(0.406)$ | $(1.633)$ | $(1.493)$ | $(0.378)$ | $(0.059)$ |
|  | 0.371 | 1.283 | 0.898 | -1.558 | -1.629 | -1.669 | 1.062 |
|  | $(0.961)$ | $(0.049)$ | $(0.434)$ | $(1.777)$ | $(1.666)$ | $(0.449)$ | $(0.062)$ |
| System | 0.399 | 0.807 | 0.829 | -1.035 | -1.050 | -0.837 | 0.637 |
|  | $(0.933)$ | $(0.525)$ | $(0.503)$ | $(2.154)$ | $(2.060)$ | $(1.180)$ | $(0.362)$ |
|  | 0.372 | 0.809 | 0.829 | -1.033 | -1.039 | -0.806 | 0.627 |
|  | $(0.963)$ | $(0.523)$ | $(0.503)$ | $(2.156)$ | $(2.077)$ | $(1.211)$ | $(0.372)$ |
|  |  |  |  |  |  |  |  |

Table 9.7. CES-translog
3. $\rho=+2.0$

|  | $\sigma_{12}$ | $\sigma_{13}$ | $\sigma_{23}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{33}$ | $\gamma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| True | 0.333 | 0.333 | 0.333 | -1.372 | -1.543 | -0.921 | 1.000 |
| MLE | 0.321 | 0.328 | 0.328 | -1.025 | -1.200 | -0.773 | 1.013 |
|  | (0.007) | (0.005) | (0.005) | (0.334) | (0.342) | (0.153) | (0.103) |
|  | 0.321 | 0.328 | 0.328 | -1.066 | -1.220 | -1.772 | 1.013 |
|  | (0.011) | (0.004) | (0.004) | (0.306) | (0.323) | (0.148) | (0.013) |
| GLS | 0.325 | 0.327 | 0.327 | -1.037 | -1.202 | -0.771 | 1.013 |
|  | (0.007) | (0.005) | (0.005) | (0.335) | (0.341) | (0.150) | (0.013) |
|  | 0.325 | 0.327 | 0.327 | -1.065 | -1.229 | -0.772 | 1.013 |
|  | (0.007) | (0.005) | (0.005) | (0.306) | (0.314) | (0.148) | (0.013) |
| Within | 0.325 | 0.327 | 0.327 | -1.016 | -1.194 | -0.766 | 1.012 |
|  | (0.008) | (0.005) | (0.005) | (0.325) | (0.331) | (0.152) | (0.013) |
|  | 0.325 | 0.327 | 0.327 | -1.047 | -1.211 | -0.7688 | 1.013 |
|  | (0.008) | (0.005) | (0.005) | (0.355) | (0.348) | (0.154) | (0.012) |
| System | 0.330 | 0.330 | 0.330 | -1.353 | -1.528 | -0.913 | 0.795 |
|  | (0.002) | (0.002) | (0.002) | (0.019) | (0.017) | (0.008) | (0.204) |
|  | 0.331 | 0.328 | 0.328 | -1.271 | -1.423 | -0.839 | 0.791 |
|  | (0.001) | (0.004) | (0.004) | (0.101) | (0.119) | (0.082) | (0.208) |

Table 9.8. Translog
3. $\rho=+2.0$

|  |  | $\sigma_{12}$ | $\sigma_{13}$ | $\sigma_{23}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{33}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| True | 0.333 | 0.333 | 0.333 | -1.372 | -1.543 | -0.921 | 1.000 |
| MLE | 0.719 | 0.723 | 0.707 | -0.773 | -0.780 | -0.703 | 1.104 |
|  | $(0.396)$ | $(0.389)$ | $(0.373)$ | $(0.599)$ | $(0.763)$ | $(0.217)$ | $(0.104)$ |
|  | 0.718 | 0.723 | 0.707 | -0.772 | -0.780 | -0.704 | 1.104 |
|  | $(0.385)$ | $(0.389)$ | $(0.374)$ | $(0.599)$ | $(0.763)$ | $(0.217)$ | $(0.104)$ |
| GLS | 0.715 | 0.720 | 0.701 | -0.848 | -0.836 | -0.759 | 1.172 |
|  | $(0.382)$ | $(0.387)$ | $(0.368)$ | $(0.524)$ | $(0.706)$ | $(0.161)$ | $(0.172)$ |
|  | 0.715 | 0.720 | 0.701 | -0.845 | -0.834 | -0.757 | 1.175 |
|  | $(0.382)$ | $(0.387)$ | $(0.368)$ | $(0.527)$ | $(0.709)$ | $(0.163)$ | $(0.175)$ |
| Within | 0.715 | 0.720 | 0.702 | -0.845 | -0.834 | -0.760 | 1.171 |
|  | $(0.382)$ | $(0.386)$ | $(0.368)$ | $(0.527)$ | $(0.709)$ | $(0.101)$ | $(0.171)$ |
|  | 0.715 | 0.719 | 0.102 | -0.842 | -0.833 | -0.760 | 1.173 |
|  | $(0.382)$ | $(0.386)$ | $(0.368)$ | $(0.530)$ | $(0.709)$ | $(0.161)$ | $(0.173)$ |
| System | 0.727 | 0.708 | 0.710 | -0.748 | -0.803 | -0.686 | 1.108 |
|  | $(0.394)$ | $(0.375)$ | $(0.377)$ | $(0.624)$ | $(0.740)$ | $(0.235)$ | $(0.108)$ |
|  | 0.727 | 0.708 | 0.711 | -0.747 | -0.803 | -0.686 | 0.108 |
|  | $(0.394)$ | $(0.375)$ | $(0.377)$ | $(0.624)$ | $(0.739)$ | $(0.234)$ | $(0.108)$ |
|  |  |  |  |  |  |  |  |

Table 9.9. Generalized-Leontief
3. $\rho=+2.0$

|  | $\sigma_{12}$ | $\sigma_{13}$ | $\sigma_{23}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{33}$ | $\gamma$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| True | 0.333 | 0.333 | 0.333 | -1.372 | -1.543 | -0.921 | 1.000 |
| MLE | 0.419 | 0.412 | 0.415 | -2.637 | -2.876 | -1.913 | 1.000 |
|  | $(0.086$ | $(0.079)$ | $(0.081)$ | $(1.264)$ | $(1.333)$ | $(0.991)$ | $(0.002)$ |
|  | 0.417 | 0.412 | 0.415 | -2.637 | -2.879 | -1.913 | 1.003 |
|  | $(0.086)$ | $(0.079)$ | $(0.082)$ | $(1.264)$ | $(1.336)$ | $(0.092)$ | $(0.003)$ |
| GLS | 0.419 | 0.412 | 0.415 | -2.637 | -2.877 | -1.914 | 1.003 |
|  | $(0.086)$ | $(0.079)$ | $(0.081)$ | $(1.266)$ | $(1.333)$ | $(0.992)$ | $(0.003)$ |
|  | 0.419 | 0.412 | 0.415 | -2.637 | -2.879 | -1.917 | 1.003 |
|  | $(0.086)$ | $(0.079)$ | $(0.081)$ | $(1.264)$ | $(1.336)$ | $(0.994)$ | $(0.003)$ |
| Within | 0.419 | 0.412 | 0.415 | -2.638 | -2.877 | -1.914 | 1.003 |
|  | $(0.086)$ | $(0.079)$ | $(0.081)$ | $(1.266)$ | $(1.333)$ | $(0.992)$ | $(0.003)$ |
|  | 0.419 | 0.412 | 0.415 | -2.639 | -2.879 | -1.918 | 1.003 |
|  | $(0.086)$ | $(0.079)$ | $(0.081)$ | $(1.263)$ | $(1.336)$ | $(0.995)$ | $(0.003)$ |
| System | 0.293 | 0.203 | 0.210 | -3.408 | -3.759 | -2.287 | 0.944 |
|  | $(0.040)$ | $(0.129)$ | $(0.122)$ | $(2.035)$ | $(2.211)$ | $(1.365)$ | $(0.055)$ |
|  | 0.293 | 0.203 | 0.211 | -3.406 | -3.761 | -2.288 | 0.944 |
|  | $(0.039)$ | $(0.014)$ | $(0.122)$ | $(2.634)$ | $(2.218)$ | $(1.367)$ | $(0.055)$ |
|  |  |  |  |  |  |  |  |

Table 9.10. CES-translog
6. $\rho=-0.85, \rho_{1}=-0.7, \rho_{2}=-0.8, \rho_{3}=-0.9$

|  | $\sigma_{12}$ | $\sigma_{13}$ | $\sigma_{23}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{33}$ | $\gamma$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| True | 2.182 | 4.364 | 6.546 | -23.59 | -17.56 | -4.156 | 0.984 |
| MLE | 1.766 | 3.945 | 5.104 | -18.80 | -12.26 | -3.145 | 1.020 |
|  | $(0.415)$ | $(0.440)$ | $(1.442)$ | $(6.788)$ | $(5.298)$ | $(1.011)$ | $(0.035)$ |
|  | 1.697 | 4.049 | 5.098 | -19.50 | -12.22 | -3.164 | 1.020 |
|  | $(0.485)$ | $(0.315)$ | $(1.448)$ | $(6.084)$ | $(5.344)$ | $(0.991)$ | $(0.035)$ |
| GLS | 1.858 | 3.824 | 5.024 | -18.42 | -12.10 | -3.106 | 1.020 |
|  | $(0.323)$ | $(0.539)$ | $(1.522)$ | $(7.163)$ | $(5.458)$ | $(1.050)$ | $(0.035)$ |
|  | 1.834 | 3.846 | 5.018 | -18.41 | -12.12 | -3.105 | 1.021 |
|  | $(0.347)$ | $(0.518)$ | $(1.528)$ | $(7.172)$ | $(5.444)$ | $(1.050)$ | $(0.036)$ |
| Within | 1.907 | 3.868 | 5.037 | -18.81 | -12.25 | -3.065 | 1.020 |
|  | $(0.274)$ | $(0.496)$ | $(1.509)$ | $(6.774)$ | $(5.314)$ | $(1.091)$ | $(0.035)$ |
|  | 1.886 | 3.877 | 5.029 | -18.84 | -12.24 | -3.069 | 1.021 |
|  | $(0.295)$ | $(0.487)$ | $(1.517)$ | $(6.751)$ | $(5.318)$ | $(1.087)$ | $(0.036)$ |
| System | 1.312 | 5.221 | 6.119 | -26.75 | -14.46 | -3.720 | 0.958 |
|  | $(0.869)$ | $(0.857)$ | $(0.451)$ | $(2.292)$ | $(3.162)$ | $(0.500)$ | $(0.026)$ |
|  | 1.186 | 5.327 | 6.176 | -26.81 | -14.513 | -3.800 | 0.958 |
|  | $(0.996)$ | $(0.965)$ | $(0.370)$ | $(1.226)$ | $(3.054)$ | $(0.356)$ | $(0.025)$ |
|  |  |  |  |  |  |  |  |

Table 9.11. Translog
4. $\rho=-0.85, \rho_{1}=-0.7, \rho_{2}=-0.8, \rho_{3}=-0.9$

|  | $\sigma_{12}$ | $\sigma_{13}$ | $\sigma_{23}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{33}$ | $\gamma$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| True | 2.182 | 4.364 | 6.546 | -25.59 | -17.56 | -4.156 | 0.984 |
| MLE | 1.778 | 4.005 | 5.125 | -18.96 | -12.28 | -3.138 | 1.018 |
|  | $(0.403)$ | $(0.359)$ | $(1.421)$ | $(6.691)$ | $(5.297)$ | $(1.017)$ | $(0.034)$ |
|  | 1.682 | 3.899 | 5.145 | -19.35 | -12.50 | -3.125 | 1.025 |
|  | $(0.499)$ | $(0.466)$ | $(1.401)$ | $(6.236)$ | $(5.065)$ | $(1.030)$ | $(1.017)$ |
| GLS | 2.197 | 3.819 | 4.806 | -19.59 | -11.90 | -2.952 | 1.019 |
|  | $(0.074)$ | $(0.545)$ | $(1.740)$ | $(5.996)$ | $(5.657)$ | $(1.204)$ | $(0.035)$ |
|  | 2.159 | 3.818 | 4.807 | -19.50 | -11.87 | -2.957 | 1.020 |
|  | $(0.022)$ | $(0.546)$ | $(1.739)$ | $(6.089)$ | $(5.688)$ | $(1.199)$ | $(0.035)$ |
| Within | 1.943 | 3.864 | 5.040 | -19.07 | -12.31 | -3.051 | 1.019 |
|  | $(0.239)$ | $(0.500)$ | $(1.506)$ | $(6.513)$ | $(5.249)$ | $(1.105)$ | $(0.034)$ |
|  | 1.928 | 3.870 | 5.027 | -19.08 | -12.29 | -3.051 | 1.019 |
|  | $(0.253)$ | $(0.493)$ | $(1.519)$ | $(6.511)$ | $(5.271)$ | $(1.105)$ | $(0.035)$ |
| System | 1.923 | 4.527 | 5.639 | -21.50 | -12.04 | -3.702 | 0.985 |
|  | $(0.686)$ | $(0.285)$ | $(0.916)$ | $(4.115)$ | $(5.522)$ | $(0.463)$ | $(0.007)$ |
|  | 2.102 | 4.688 | 5.491 | -22.43 | -11.96 | -3.798 | 0.987 |
|  | $(0.079)$ | $(0.324)$ | $(1.055)$ | $(3.157)$ | $(5.601)$ | $(0.358)$ | $(0.002)$ |
|  |  |  |  |  |  |  |  |

Table 9.12. Generalized-Leontief
4. $\rho=-0.85, \rho_{1}=-0.7, \rho_{2}=-0.8, \rho_{3}=-0.9$

|  | $\sigma_{12}$ | $\sigma_{13}$ | $\sigma_{23}$ | $\sigma_{11}$ | $\sigma_{22}$ | $\sigma_{33}$ | $\gamma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| True | 2.182 | 4.364 | 6.546 | -25.59 | -17.56 | -4.156 | 0.984 |
| MLE | -1.751 | 3.270 | 3.184 | -5.611 | -2.647 | -0.705 | 0.703 |
|  | (2.933) | (1.093) | (3.362) | (19.98) | (14.91) | (3.451) | (0.057) |
|  | -0.883 | 2.904 | 2.849 | -4.656 | -2.177 | -0.630 | 0.703 |
|  | (3.065) | (1.459) | (3.697) | (20.93) | (15.38) | (3.526) | (0.281) |
| GLS | -1.847 | 4.087 | 3.812 | -6.306 | -2.463 | -0.692 | 1.030 |
|  | (4.029) | (0.277) | (2.734) | (19.28) | (15.10) | (3.464) | (0.046) |
|  | -1.848 | 4.080 | 3.820 | -6.305 | -2.463 | 0.692 | 1.030 |
|  | (4.030) | (0.283) | (2.726) | (19.28) | (15.10) | (3.464) | (0.045) |
| Within | -1.977 | 4.120 | 3.835 | -6.258 | -2.460 | -0.693 | 1.030 |
|  | (4.133) | (0.240) | (2.723) | (19.33) | (15.10) | (3.462) | (0.046) |
|  | -1.977 | 4.120 | 3.835 | -6.258 | -2.460 | -0.693 | 1.030 |
|  | (4.240) | (0.407) | (2.955) | (34.92) | (29.11) | (8.298) | (0.048) |
| System | -0.053 | 3.955 | 4.623 | -4.013 | -1.966 | -0.758 | 1.015 |
|  | (2.236) | (0.408) | (1.923) | (21.57) | (15.60) | (3.398) | (0.030) |
|  | 0.068 | 3.916 | 4.633 | -4.008 | -1.967 | -0.755 | 1.015 |
|  | (2.114) | (0.448) | (1.913) | (21.53) | (15.60) | (3.401) | (0.030) |

Table 10. Concavity Tests From the Flexible Cost Functions ${ }^{\text {a }}$

| Experiment | CES-translog | Translog | Generalized-Leontief |
| :--- | :--- | :--- | :--- |
| 1. $\rho=-0.67$ | $-9.2541^{\mathrm{b}}$ | -8.9012 | -4.8521 |
|  | $-7.2638(42 / 50)^{\mathrm{c}}$ | $-7.8210(0 / 50)$ | $-3.0543(0 / 50)$ |
|  | -0.0200 | +0.4210 | +3.4243 |
| 2. $\rho=-0.5$ | -5.9263 | -6.2397 | -3.9213 |
|  | $-5.4921(50 / 50)$ | $-5.4238(0 / 50)$ | $-2.5027(0 / 50)$ |
|  | -0.0472 | +0.1431 | +1.8217 |
| 3. $\rho=-0.25$ | -3.2641 | -3.2543 | -3.2012 |
|  | $-2.3591(48 / 50)$ | $-2.3412(50 / 50)$ | $-1.9201(30 / 50)$ |
|  | -0.0212 | -0.0021 | -0.2082 |
| 4. $\rho=+0.1$ | -3.0321 | -3.0271 | -2.4176 |
|  | $-2.2100(50 / 50)$ | $-2.5645(50 / 50)$ | $-2.3579(0 / 50)$ |
|  | -0.0067 | -0.0023 | +0.2067 |
| 5. $\rho=+2.0$ | -1.5792 | -1.5532 | -3.2088 |
|  | $-1.3232(50 / 50)$ | $-1.4721(0 / 50)$ | $-2.7265(50 / 50)$ |
|  | -0.3211 | +0.6212 | -1.4991 |
| 6. $\rho=-0.45$ | -2.2512 | -2.0812 | -17.231 |
| $\rho_{1}=-0.4$ | $-1.9920(50 / 50)$ | $-1.9632(50 / 50)$ | $-2.1392(0 / 50)$ |
| $\rho_{2}=-0.5$ | -0.0821 | -0.0133 | +0.1789 |
| $\rho_{3}=-0.6$ |  |  |  |
| 7. $\rho=-0.85$ | -19.291 | -20.124 | -9.2344 |
| $\rho_{1}=-0.7$ | $-14.482(0 / 50)$ | $-14.234(0 / 50)$ | $-2.1245(0 / 50)$ |
| $\rho_{2}=-0.8$ | +0.1421 | +0.1235 | +2.5621 |
| $\rho_{3}=-0.9$ |  |  |  |

${ }^{\text {a }}$ The four estimators lead to similar very results. These are from the within estimator.
${ }^{6}$ The three entries are eigen values for the matrix of $\sigma_{i j}$ 's.
${ }^{\circ}$ The values in parentheses indicate the number of successes (of the concavity condition) among the 50 iterations.

## 6. Conclusions

We have considered the estimation of firm-specific technical inefficiency using stochastic frontier models with panel data and have measured characteristics of the underlying technologies. Several findings are worth mentioning. When the underlying technology is very simple (that is, Cobb-Douglas or CES), the ability of the stochastic frontier models to estimate firm-specific inefficiency is very good, regardless of the choice of functional forms. As the complexity of the underlying technology (that is, CRESH production technology) increases, however, this ability markedly decreases. The performance of estimators is similar with respect to the estimation of firm-level inefficiency and the underlying technology. Because
of weak assumptions and computational ease, we prefer the within estimator to mle and gls. Thus we ameliorate two common problems of stochastic frontiers. We do not have to impose a strong (and arbitrary) structure for the distribution of technical inefficiency in order to separate noise and inefficiency. We also do not need the independence of technical inefficiency and inputs. A minor finding is that the CES-TL often is better than the TL in modelling the underlying technology. Specifically, for an extreme case-very easy of difficult substitution-the CES-TL is superior to the TL. In the concavity test of flexible functional forms, the CES-TL dominates the TL and the GL over a wide range of technologies.

One direction for future research is to compare the stochastic frontier models with Data Envelopment Analysis (DEA) in the estimation of firm-specific technical inefficiency. With the same known underlying technology (the CRESH production function), we can use DEA [Charnes, Cooper and Rhodes 1978] and its modifications [Charnes, et al. 1985]. To our knowledge this would be the first study to analyze the relative merits of these competing methodologies in such a general experimental setting. ${ }^{17}$

## Notes

1. Farrell's idea is to construct a reference production set and measure productive efficiency relative to it. See Kopp [1981], Kopp and Diewert [1982] and Charnes and Cooper [1985].
2. For a treatment of the differences between an average production function from a frontier production function, see Frrsund et al. [1977, 1980].
3. Traditional microeconomics gives little attention to the presence of productive inefficiency. See Färe, Grosskopf and Lovell [1985] for an excellent exception.
4. Another drawback is that stochastic frontier models have difficulty in modelling multi-output technologies.
5. Farrell questioned whether allocative efficiency was of any intrinsic usefulness. Instead of allocative inefficiency, he argues that the measurement of technical efficiency was of primary concern.
6. For the specification of alternative modeling scenarios for production processes see Judge et al. [1985].
7. Reviews of panel data models can be found in Hsiao [1986], and Chamberlain [1983].
8. For a Fourier expansion as the approximating form, see Gallant [1981].
9. $\sigma_{i j}(x)=\frac{\sum x_{k} f_{k}}{x_{i} x_{j}} \cdot \frac{F_{i j}}{F} ;-\infty<\sigma_{i j}<+\infty$,
where
$F_{i j}$ is the co-factor of $f_{i j}$ in $F$,
$F$ is $\left[\begin{array}{cccc}0 & f_{1} & \ldots & f_{n} \\ f_{l} & f_{11} & \ldots & f_{1 n} \\ \cdot & & \vdots \\ \cdot & f_{22} & \vdots \\ \cdot & & \ddots & \\ f_{n} & f_{n 1} \ldots & f_{n n}\end{array}\right]$.
10. This characteristic of firm-specific inefficiency was recently explored by Seale [1985].
[^2]
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[^0]:    *The refereeing process of this paper was handled through J. van den Broeck.

[^1]:    ${ }^{\text {a }}$ Each column/row entry is the correlation coefficient of firm-specific technical inefficiency in terms of excess cost between a true firm-specific technical inefficiency and an estimated firm-specific inefficiency.

[^2]:    11. Aigner, Lovell and Schmidt [1977] assumed that $u$ has either a half-normal density or an exponential density. They implicitly assumed that the likelihood of inefficiency behavior monotonically decreases for increasing levels of inefficiency. But Stevenson [1981] suggested that the possibility of a non-zero mode for the density function of $u$ would seem a more tenable assumption (that is, a truncated normal distribution with a non-zero mean). In the case of a half-normal distribution, $u$ is the absolute value of a variable distributed as $N\left(0, \sigma_{\omega}^{2}\right)$. Its mean is $\left.\sqrt{\left(2 / \pi \cdot \sigma_{\omega}\right.}\right)$ and its variance is $[(\pi-2)$ / $\pi\} \cdot \sigma_{u}^{2}$
    12. For general discussions of duality theory, see Diewert [1971, 1973] and Blackorby et al. [1978].
    13. Examples of mle in previous stochastic frontier studies include Afriat [1972], Aigner, Amemiya, Poirier [1976], Meeusen and van den Broeck [1977], Aigner, Lovell and Schmidt [1977] and Greene [1980a, b].
    14. For more detailed explanations about the construction of the firm-specific technical inefficiency index, see Schmidt and Sickles [1984].
    15. Instead of the magnitude of $\sigma_{\mathrm{u}}^{2}$ and $\sigma_{\mathrm{v}}^{2}$, their relative size is important: (1) $\lambda=\sigma_{\mathrm{u}}^{2} / \sigma_{\mathrm{v}}^{2}=\infty$ is the full frontier case for which there exists no statistical noise and (2) $\lambda=0$ in which case no technical inefficiency exists.
    16. The functional forms with a self-dual nature are limited (for example, the Cobb-Douglas and the CES production forms), and thus this approach cannot be applied to complex production technologies.
    17. For the comparison of stochastic frontier models and DEA, see Gong [1987].
